| Classical | Quantum |
| --- | --- |
| $I(A:B)$ | $\boxed{I_{A:B}(\rho) = S(\rho_A) + S(\rho_B) - S(\rho)}$ mutual info. |
| $H(B\mid A=i)$ | $S(\rho_{B\mid i})$ with $\rho_{B\mid i}$ = post-measurement state of B conditionned to the outcome $i$ occuring with proba $p_i = \mathrm{tr}(\Pi_i^A \otimes 1\, \rho)$, $\{\Pi_i^A\} = \perp$ projectors of $\mathcal{H}_A$ $(\Pi_i^A \Pi_k^A = \delta_{ik} \Pi_i^A)$ <br> $\rho_{B\mid i} = \dfrac{\mathrm{tr}_A(\Pi_i^A \otimes 1\, \rho)}{p_i}$ |
| $H(B) - H(B\mid A)$ | $\boxed{J_{B\mid A}(\rho) = \max_{\{\Pi_i^A\}} \left\{ S(\rho_B) - \sum_i p_i\, S(\rho_{B\mid i}) \right\} = \max_{\{\Pi_i^A\}} I_{A:B}(\mathcal{M}_A^{\Pi}(\rho))}$ <br> $\mathcal{M}_A^{\Pi}(\rho) = \sum_i \Pi_i^A \otimes 1\, \rho\, \Pi_i^A \otimes 1$ = CP trace-preserving map of measurement |

• In general, in the quantum case $I_{A:B}(\rho) \neq J_{B|A}(\rho)$

**Def. 3** (OLLIVIER-ZUREK, HENDERSON-VEDRAL '01)

$$\boxed{\text{Quantum discord } \delta_A(\rho) = \underbrace{I_{A:B}(\rho)}_{\boxed{\text{TOTAL CORRELATIONS}}} - \underbrace{J_{B|A}(\rho)}_{\boxed{\text{"CLASSICAL" CORRELATIONS}}}}$$

**Prop:**

(i) $\delta_A(|\Psi\rangle) = E_{EoF}(|\Psi\rangle) = S(\rho_A)$

(ii) $\delta_A(\rho) \geqslant 0$

(iii) $\delta_A(\rho) = 0 \Leftrightarrow \rho \in \mathcal{C}_A = \left\{ \rho = \sum_i q_i |\alpha_i\rangle\langle\alpha_i| \otimes \rho_{B|i} \,;\ \{|\alpha_i\rangle\} \text{ ONB of } \mathcal{H}_A,\ q_i \geqslant 0,\ \sum_i q_i = 1 \right\}$

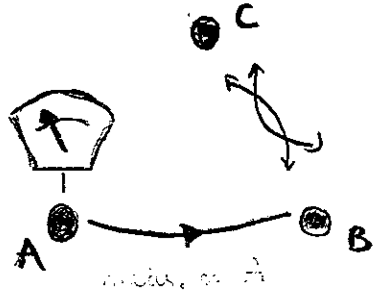
"A-classical states"

NB: $\delta_A$ not symmetric under $A \leftrightarrow B$

- Relation between $\delta_A$ and $E_{EoF}$ :

Prop: $\boxed{E_{EoF}(\rho_{BC}) = \delta_A(\rho_{AB}) + S(\rho_{AB}) - S(\rho_A)}$ if ABC is in a pure state

(KOASHI - WINTER '04)

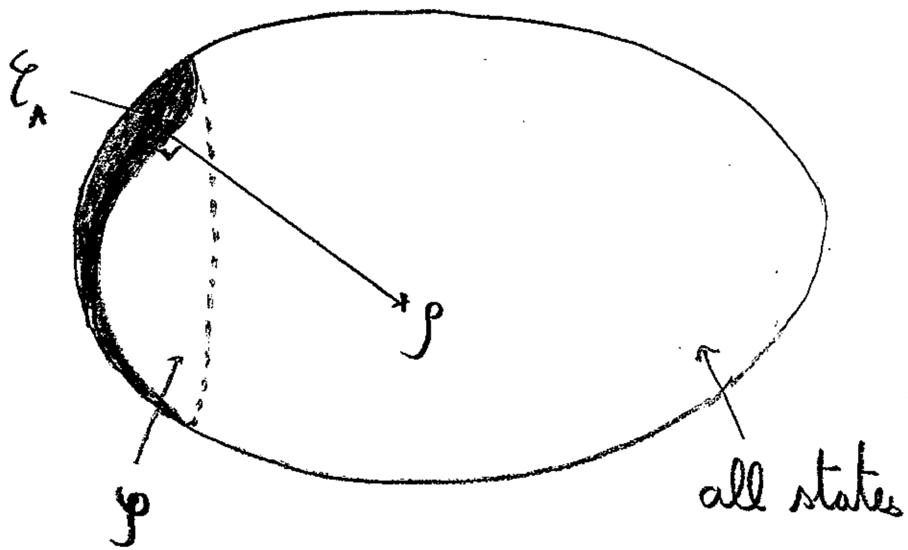



- Geometric quantum discord

$$\boxed{Q_A(\rho) = d_B(\rho, \mathcal{C}_A)^2}$$

Clearly $*$ $0 \leqslant E(\rho) \leqslant Q_A(\rho)$

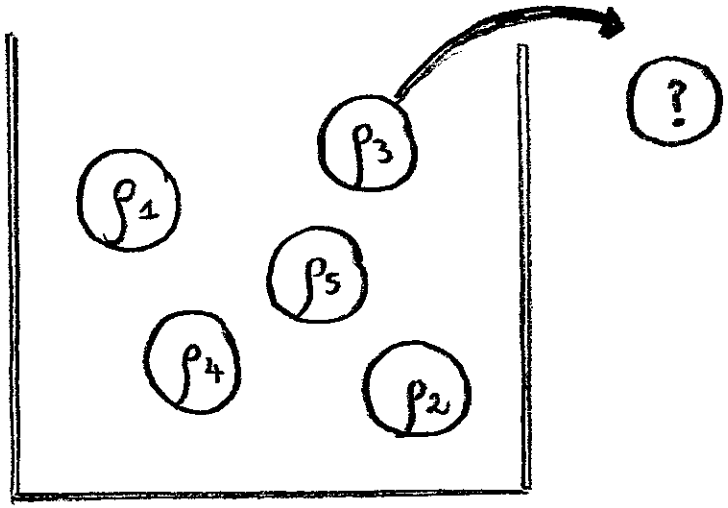
$*$ $Q_A(|\Psi\rangle) = E(|\Psi\rangle)$




NB : $\mathcal{C}_A$ not convex ( its convex hull is $\mathcal{S}$ )

$\Rightarrow$ no convex roof construction for $Q_A$

# QUANTUM STATE DISCRIMINATION




* A receiver is given a <u>random state</u> $\rho_i$, chosen among a known set $\{\rho_1, \dots, \rho_n\}$, <u>with prior probability</u> $\eta_i$.

* His task is to identify which state he has received by performing a measurement.

* One looks for the <u>optimal generalized measurement</u> (POVM $\{M_i\}_{i=1}^{n}$), depending on $\{\rho_1, \eta_1; \dots, \rho_n, \eta_n\}$, which yields the maximal success proba

$$\boxed{P_S^{opt}(\{\rho_i, \eta_i\}) = \max_{\{M_i\}\ \text{POVM}} \left\{ \sum_i \eta_i\, \mathrm{tr}(\rho_i M_i) \right\}}$$

PROBA OF MEAS. OUTCOME "$i$" GIVEN THAT THE STATE IS $\rho_i$

Prop (D.S. - ORSZAG '13)

- $D_A(\rho) = 2\left(1 - \sqrt{F_A(\rho)}\right)$ where the maximal fidelity $F_A(\rho) \equiv \max_{\sigma \in \mathcal{C}_A} F(\rho, \sigma)$ to a A-classical state is the maximal success probability

$$\boxed{F_A(\rho) = \max_{\{|\alpha_i\rangle\} \text{ ONB of } \mathcal{H}_A} P_S^{opt}(\{\rho_i, \eta_i\})}$$ with

$$\rho_i = \eta_i^{-1} \sqrt{\rho}\, |\alpha_i\rangle\langle\alpha_i| \otimes 1_B \sqrt{\rho} \;, \quad \eta_i = \langle\alpha_i|\rho_A|\alpha_i\rangle$$

- Moreover, the closest A-classical states to $\rho$ are

$$\sigma_\rho = \frac{1}{F_A(\rho)} \sum_{i=1}^{n} |\alpha_i^{opt}\rangle\langle\alpha_i^{opt}| \otimes \langle\alpha_i^{opt}|\sqrt{\rho}\, \Pi_i^{opt} \sqrt{\rho}\, |\alpha_i^{opt}\rangle_A$$

ONB of $\mathcal{H}_A$ maximizing $P_S^{opt}(\{\rho_i, \eta_i\})$

optimal projective measurements for discriminating the $\rho_i$'s