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Quantum Dynamical Semigroups for Entangled Pure State Preparation

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In collaboration with L. Viola, Dartmouth College,
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[T-Viola, Phil. Trans. A, 2012; T-Viola, arXiv:1304.4270]

Disclaimer

- I am not a mathematical physicist!
- I am not a physicist!

So what am I? Why am I here?

- I am not a philosopher either (phew!)

I do research as a quantum "control engineer"
...dynamical systems, probability & stochastic processes, optimization, estimation...

Language may be a bit different!
I will try to keep technicalities to a minimum.
Stop me and ask!
Other viewpoints and comments are welcome.

The Control Theorist's Recipe

(aka this talk's outline)

- **Main Ingredients:**

  1. **A control task and/or performance index; Possibly useful/interesting!**

  2. **A class of dynamical systems, with inputs or tunable parameters;**

  3. **Constraints on the admissible controls;**

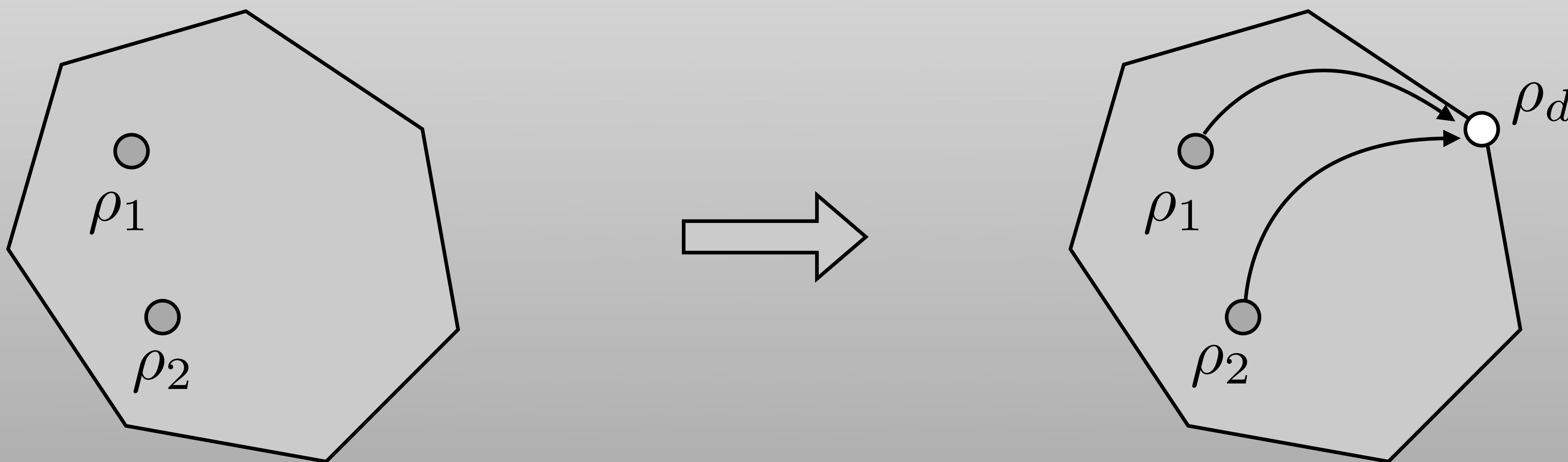
  4. **A grain of salt and a bit of luck;**

- **Cooking Directions:**
  Put everything together, do some linear algebra with it, and shake well!

Pure State Preparation for Quantum Systems

Consider a *finite-dimensional* quantum system with Hilbert space $\mathcal{H}$
General states (*density operators*) form a convex set,
extreme points are **pure states** (*rank-one orth. projections*):

$$\rho \in \mathfrak{D}(\mathcal{H}) := \{\rho = \rho^\dagger > 0, \; \mathrm{trace}(\rho) = 1\}$$



Task:

Prepare a *given pure state* irrespective of the initial one,
i.e. make it **Global Asymptotycally Stable (GAS)**

Why Pure State Preparation? Which ones?

- **The problem is key to**:
  I. **Cooling to minimum energy eigenstate;**
  (nanomechanical, opto-electro-mechanical systems)

  II. **Initialization for other control algorithms;**
  (optimal, adiabatic)

  III. **Quantum Information Processing:**
  a. Initialization of the "quantum register" for an algorithm
  (Basic Di Vincenzo's requirement for a Q Computer);
  Initialization in a protected quantum code;
  b. Initialization of an **entangled state** for one-way quantum computation;
  c. Reset a quantum memory;
  d. Create steady **entangled states** (entanglement on demand);

Big(ger) Picture: Open Systems and Control

➡ Need for accurate control the dynamics of realistic quantum open systems.

- **Fundamental & Practical Significance:**

  ✓ ***Rigorous (and useful) system-theoretic framework for open systems;***

  ✓ Robust realization of Quantum Information Processing (QIP);

  ✓ Enhanced sensitivity in precision measurements & metrology;

  ✓ Engineer new “forms”/phases of matter;

- **Two Prevailing & Complementary Approaches:**

I. **Environment as Enemy: we want to “remove” the coupling.**
Noise suppression methods, active and passive, including *hardware engineering, noiseless subsystems, quantum error correction, dynamical decoupling;*

II. **Environment as Resource: we want to “engineer” the coupling.**
Needed for state preparation, open-system simulation, and much more...

Dissipation for Information Engineering

- **Experiments**

  Open Sys. Simulator

ARTICLE

doi:10.1038/nature09801

An open-system quantum simulator with trapped ions

Julio T. Barreiro[1]*, Markus Müller[2,3]*, Philipp Schindler[1], Daniel Nigg[1], Thomas Monz[1], Michael Chwalla[1,2], Markus Hennrich[1], Christian F. Roos[1,2], Peter Zoller[2,3] & Rainer Blatt[1,2]

  Entanglement Generation

PRL 107, 080503 (2011) PHYSICAL REVIEW LETTERS week ending 19 AUGUST 2011

Entanglement Generated by Dissipation and Steady State Entanglement of Two Macroscopic Objects

Hanna Krauter,[1] Christine A. Muschik,[2] Kasper Jensen,[1] Wojciech Wasilewski,[1,*] Jonas M. Petersen,[1] J. Ignacio Cirac,[2] and Eugene S. Polzik[1,†]

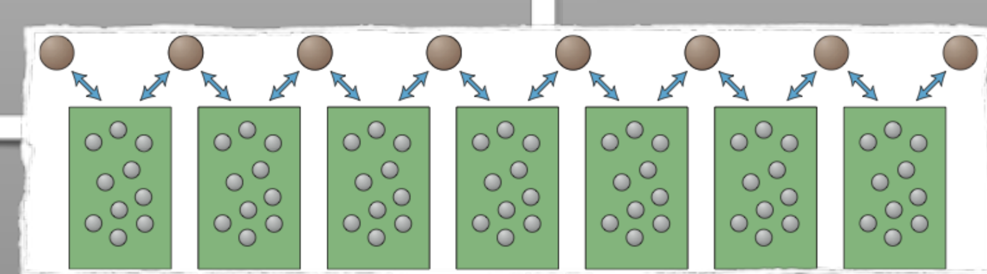
- **Theory**

  Dissipation for QIP

nature physics LETTERS
PUBLISHED ONLINE: 20 JULY 2009 | DOI: 10.1038/NPHYS1342

Quantum computation and quantum-state engineering driven by dissipation

Frank Verstraete[1]*, Michael M. Wolf[2] and J. Ignacio Cirac[3]*



Quantum Dynamical Semigroups

- System's Hamiltonian dynamics alone is not enough: *unitary, not contractive*; we need dissipation, interaction with an environment/measurement apparatus;

- Assume the environment to be Markovian, it then yields a Markovian Master Equation/QDS generator:

[Gorini-Kossakovski-Sudarshan/Lindblad, 1974]

$$\dot{\rho}_t = \mathcal{L}(\rho) = -i[H, \rho_t] + \sum_{k=1}^{p} L_k \rho_t L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k,\ \rho_t\}$$

Hamiltonian part

$$H = H^\dagger, \quad L_k \in \mathbb{C}^{n \times n}.$$

Dissipative, "noisy" part

- **good:** linear and easy to study - exponential convergence;
- **bad:** representation of $\mathcal{L}$ in terms of Hamiltonian and noise is not unique.

We will assume we can, under some *constraints*, engineer these operators

$$\mathcal{L}' = \mathcal{L} \iff L'_k = L_k + \lambda_k I \quad H' = H + \frac{i}{2}\sum_k (L_k \lambda_k^* - \lambda_k L_k^\dagger)$$

Can I use QDS to prepare a desired pure state?

State Engineering Methods

[T-Viola, *Automatica, 2009;*
T-Schirmer-Wang, *IEEE T.A.C.*, 2010;]

Yes, but...

- **Fact:** A QDS that does the job always exists, and it can be very “simple” :
  - **Pure state: a single L is enough, with ladder-type operator;** [T-Viola, IEEE T.A.C., 2008].
  - **Mixed state: just H and a single L (tri-diagonal matrices);** [T-Schirmer-Wang, IEEE T.A.C., 2010]
- **Can I engineer it with experimentally-available, constrained control capabilities? Typically not. We need to specify the control type:**
  - **Open-loop** Hamiltonian or noise-operator switching control;
  - Output-**feedback** for a fixed measurement;
  - Filtering-based **feedback**;

...and the type of constraints (CRITICAL):

- Structure, speed and amplitude of the control actions;
- **LOCALITY constraints.**

key limitation for entanglement generation

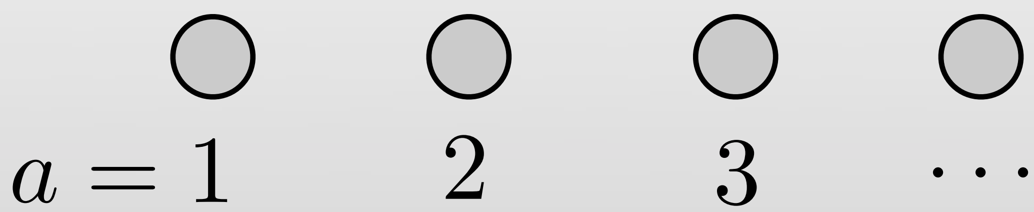
Can I use constrained QDS
to prepare entangled pure states?

State engineering under locality constraints

[T-Viola, *Phil. Trans. R. Soc. A, 2012; T-Viola, arXiv:1304.4270*]

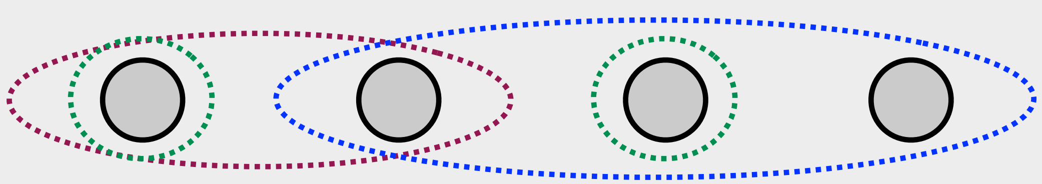
Multipartite Systems and Locality Constraints

- Consider *n* finite-dimensional systems:



$$\mathcal{H}_{\mathcal{Q}} = \bigotimes_{a=1}^{n} \mathcal{H}_a$$

- **Locality notion:** start by specifying from the beginning *subsets of indexes,* or ***neighborhoods,*** corresponding to group of subsystems:



$$\mathcal{N}_1 = \{1, 2\}$$
$$\mathcal{N}_2 = \{1, 3\}$$
$$\mathcal{N}_3 = \{2, 3, 4\}$$

...on which "we can act simultaneously": how?

- A *noise operator* is said Quasi-Local (QL) if $L_k = L_{\mathcal{N}_k} \otimes I_{\bar{\mathcal{N}}_k}$
- A *Hamiltonian* is said QL if:

$$H = \sum_k H_k, \quad H_k = H_{\mathcal{N}_k} \otimes I_{\bar{\mathcal{N}}_k}$$

Note: definitions depend on the neighborhoods!

This framework encompasses different notions: graph-induced locality, N-body locality, etc...

Quasi-Local Stabilizable State with Dissipation

- **Let us start with a simple case: no drift, no Hamiltonian, noise only.**

- **Definition:** A state $\rho = |\psi\rangle\langle\psi|$ is **Dissipatively Quasi-Local Stabilizable (DQLS)** if there exist a QDS generator $\mathcal{L}$:

1. associated to $H = 0$ and **QL** noise operators $\{D_k\}$, s. t. $D_k|\psi\rangle = 0$;

2. for which ρ is GAS.

- **Which states are DQLS? Is there a way to test a target state?**

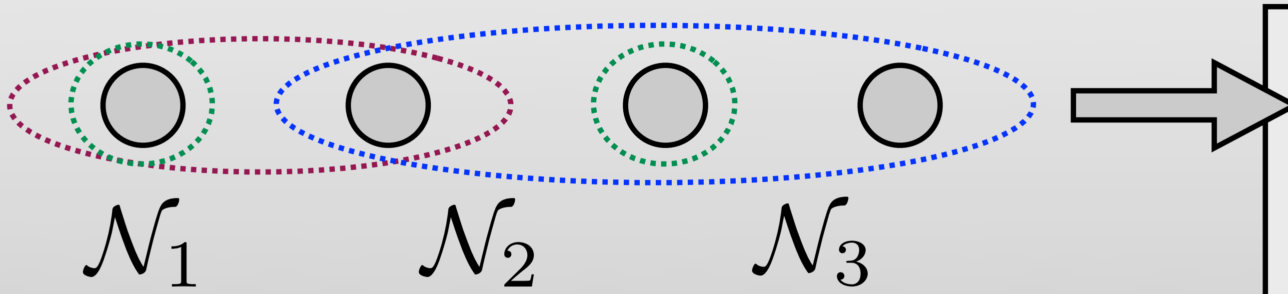
- Locality-constrained, engineed noise operators.
- For now, assume there is no free dynamics. We will bring it into the picture later.

$D_k|\psi\rangle = 0$

is a technical assumption, in order to make the decomposition between Hamiltonian and noise part unique.

Test and Characterization of DQLS

- For each *neighborhood* compute the reduced states;



$$\rho_{\mathcal{N}_1},\ \rho_{\mathcal{N}_2},\ \rho_{\mathcal{N}_3}$$
$$\rho_{\mathcal{N}_k} = \mathrm{trace}_{\bar{\mathcal{N}}_k}(\rho),$$

- For each neighborhood calculate the *range* of the reduced state tensor the identity on the rest: $$\mathcal{H}_{\mathcal{N}_k} = \mathrm{range}(\rho_{\mathcal{N}_k} \otimes I_{\bar{\mathcal{N}}_k})$$

- ***Theorem:***

$$\mathcal{H}_0 := \bigcap_k \mathcal{H}_{\mathcal{N}_k} = \mathrm{range}(\rho)$$

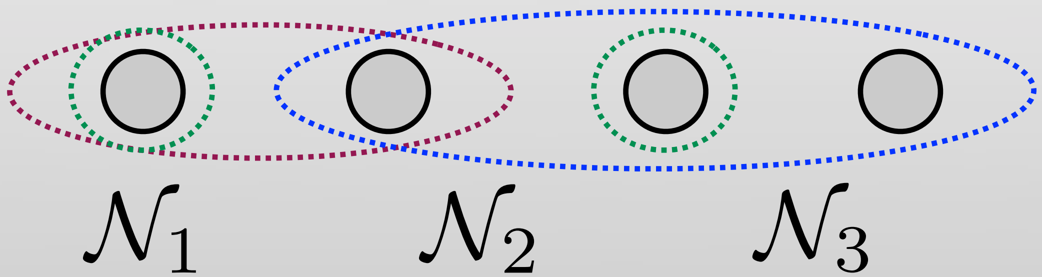
if and only if ρ **is DQLS;**

Provides a simple, easily automated test with only two inputs: the state and the neighborhoods

Can be stabilized by QL $\{D_k\}$ that make $\{\mathcal{H}_{\mathcal{N}_k}\}$ invariant (necessary condition) and attractive.

Idea of the proof

- We know how to design QL QDSs stabilize given subspaces **locally**;
- For each *neighborhood* we can **“stabilize the range”** of the reduced state;



$$\rho_{\mathcal{N}_1},\ \rho_{\mathcal{N}_2},\ \rho_{\mathcal{N}_3}$$

$$\rho_{\mathcal{N}_k} = \mathrm{trace}_{\bar{\mathcal{N}}_k}(\rho),$$

i.e. stabilize the states with range in: $\mathcal{H}_{\mathcal{N}_k} = \mathrm{range}(\rho_{\mathcal{N}_k} \otimes I_{\bar{\mathcal{N}}_k})$

- **IF part:** By analysis of the invariant sets with Lyapunov techniques. The intersection of the QL stable set is the largest invariant set and contains the GAS set. Hence, target is GAS if it is the unique set in the intersection, or:

$$\mathrm{range}(\rho) = \bigcap_k \mathcal{H}_{\mathcal{N}_k} := \mathcal{H}_0$$

- **ONLY IF part, less obvious:** show (using contraction properties) that **each reduced state must be invariant for the local part of the generator** in order to stabilize the target.

DQLS, Or Not? Physical Interpretation

- **Equivalent characterization:** $\rho = |\psi\rangle\langle\psi|$ is DQLS **if and only if** it is the unique, frustration-free ground state of a QL Hamiltonian, that is:

- There exists a QL Hamiltonian for which $|\psi\rangle$ is the unique ground state and

$$H = \sum_k H_k, \quad H_k = H_{\mathcal{N}_k} \otimes I_{\bar{\mathcal{N}}_k}$$

such that $\langle\psi|H_k|\psi\rangle = \min \sigma(H_k), \quad \forall k.$

Proof: It suffices to choose $H_k = \Pi^{\perp}_{\mathcal{N}_k} \otimes I_{\bar{\mathcal{N}}_k}$, $\Pi^{\perp}_{\mathcal{N}_k}$ projects on $\mathrm{supp}(\rho_{\mathcal{N}_k})^{\perp}$.

- Interesting connection to physically-relevant cases, and previous work by Verstraete, Perez-Garcia, Cirac, Wolf, B. Kraus, Zoller and co-workers.
- **Differences:**
  In their setting, the proper locality notion is induced by the target state itself.
  In our setting, *the locality is fixed a priori. We also prove necessity of the condition.*

Is Dissipation Enough?

- **Which states are DQLS? Using our test, it turns out that...**
  - *GHZ states (maximally entangled) and W states are never DQLS*;
  - *Any graph state is DQLS* with respect to **the locality induced by the graph;**
    To each node is assigned a neighborhood, which contain all the nodes connected by edges.
  - *Generic MPS/PEPS are QLS* for **some locality definition**...

    `[see work by Peres-Garcia, Wolf, Cirac and co-workers]`

Good, but not great. Important states are left out. Can we do better?

So far we have not included the Hamiltonian in the tunable parameters.

- **Definition:** A state $\rho = |\psi\rangle\langle\psi|$ is **Quasi-Local Stabilizable (QLS)** if there exist a QDS generator $\mathcal{L}$ associated to QL operators $H, \{D_k\}$ for which ρ is GAS.

Which states are QLS? Is DQLS equivalent to QLS?

QLS states are more than DQLS states!

- **GHZ states are never DQLS, but are QLS for some locality notions:**

$$\rho_{\mathrm{GHZ}} = |\Psi\rangle\langle\Psi|, \quad |\Psi\rangle \equiv |\Psi_{\mathrm{GHZ}}\rangle = (|000\ldots0\rangle + |111\ldots1\rangle)/\sqrt{2}.$$

By symmetry considerations $\mathcal{H}_0$ (intersection of supports of reduced states) is two-dimensional and contains all superpositions of $|000\ldots0\rangle, |111\ldots1\rangle$.

- *Necessary condition for DQLS:*
Dissipation alone cannot do anything more than making $\mathcal{H}_0$ attractive!

- *We need to "destabilize" the others!*
If we can construct a QL Hamiltonian such that:

$$H|000\ldots0\rangle = |1\ldots10\ldots0\rangle - |0\ldots01\ldots1\rangle)/\sqrt{2},$$
$$H|111\ldots1\rangle = -|1\ldots10\ldots0\rangle + |0\ldots01\ldots1\rangle)/\sqrt{2},$$

then the target is stable, but the other state is pushed out of invariant set:

$$H|\Psi_{\mathrm{GHZ}}\rangle = 0, \quad H|\Psi_{\mathrm{GHZ}}^{\perp}\rangle = \frac{2}{\sqrt{2}}(|1\ldots10\ldots0\rangle - |0\ldots01\ldots1\rangle) \notin \mathcal{H}_0.$$

QLS states are more than DQLS states!

- $$\rho_{\text{GHZ}} = |\Psi\rangle\langle\Psi|, \quad |\Psi\rangle \equiv |\Psi_{\text{GHZ}}\rangle = (|000\ldots0\rangle + |111\ldots1\rangle)/\sqrt{2}.$$

These Hamiltonians do exist: for example, pick a partition in two halves of the qubits, call the subsets S_1, S_2 :

$$H = (\bigotimes_{j\in S_1} \sigma_x^{(j)}) \otimes I_{S_2} - I_{S_1} \otimes (\bigotimes_{k\in S_2} \sigma_x^{(k)})$$

Unfortunately, it can be proven that the neighborhoods *must* contain half of the qubits! GHZ are hard to prepare!

then the target is still stable, and the other state is not.

Similarly, W states can be stabilized with 2-body QL dissipation and Hamiltonians (they were not DQLS).

Message: QL dissipation and Hamiltonian actions are intrinsically different and their interaction can be useful!

Necessary Conditions for a QLS Hamiltonian

- **Generalizing the example:**

  In order to be **QLS**, $\rho = |\Psi\rangle\langle\Psi|$ must be in the kernel of the generator. It follows that (up to a transformation in standard form):

  - $|\Psi\rangle$ **must be in the kernel of each noise operator;** *(same condition of the DQLS case)*

  - $|\Psi\rangle$ **must be invariant for the Hamiltonian;**

  - $\mathcal{H}_0$ **must not be invariant for the Hamiltonian;**

  These are necessary conditions. How to proceed?
  Any way to check if such an Hamiltonian exists?
  We need another point of approach...

Useful Block-Matrix Representation

An orthogonal subspace decomposition induces matrix decomposition(s)

$$\mathcal{H} = \mathcal{H}_S \oplus \mathcal{H}_R \quad \Longrightarrow \quad X = \left(\begin{array}{c|c} X_S & X_P \\ \hline X_Q & X_R \end{array}\right)$$

Choose $\mathcal{H}_S = \mathrm{range}(\rho)$ such that our target pure state is in the form:

$$\rho = \left(\begin{array}{c|c} \rho_S & 0 \\ \hline 0 & 0 \end{array}\right),$$

What are the properties of $\mathcal{L}$ that ensure $\mathcal{L}(\rho) = 0$?

Invariance Conditions

- By direct computation, the state ("subspace") is **invariant** *if and only if:*

$$L_k = \left(\begin{array}{c|c} L_{S,k} & L_{P,k} \\ \hline 0 & L_{R,k} \end{array} \right) \quad \forall k,$$

$$iH_P - \frac{1}{2}\sum_k (L^\dagger_{S,k} L_{P,k}) = 0,$$

quadratic "fine-tuning": overlooked for a while...

- If we require (our technical assumption to fix *one representation*):

$$\rho = |\psi\rangle\langle\psi| \qquad L_k|\psi\rangle = 0$$

then it must be $L_{S,k} = 0$; Conditions become linear!

Warning: only true for pure states

Randomized Testing and Construction of QDS

- Assume we can parametrize the available family of QDS in a set of parameters, e.g. with respect to an operator basis $\{B_j\}$:

$$H = \sum_j \alpha_j B_j \quad L_k = \sum_j \beta_{k,j} B_j \qquad |\alpha_j, \beta_{kj}| \leq \gamma_j, \gamma_j > 0$$

- The **locality constraints** are linear (e.g. choose a factorized basis);
- The **invariance constraints** are (in a standard form) linear $\mathcal{L}(\rho) = 0$;
- The available set is associated to the intersection with an (lower-dimensional) hyperplane. Re-parametrize it with α'_j, β'_{kj} ;

  We can prove:

Fix what you need, and shake the rest

- ***Theorem, and algorithm for testing:***
  > Impose the locality and invariant constraints.
  > Fix a centered interval for α'_j, β'_{kj} , and pick them at random with uniform density.
  If the target state is QLS subject to the constraints,
  then it is made GAS by this choice with probability 1 (easy to check).

Idea of Proof: Dissipation-Induced Decomposition

- Construct DID: $\mathcal{H} = \mathcal{H}_S \oplus \mathcal{H}_T^{(1)} \oplus \ldots \oplus \mathcal{H}_T^{(d)} \oplus \mathcal{H}_R^{(d)}$

- Given invariance of target, the noise operators can be put in standard matrix form...

$$L_k = \left[\begin{array}{c|cccc} L_S & \hat{L}_P^{(0)} & 0 & \cdots & \\ \hline 0 & L_T^{(1)} & \hat{L}_P^{(1)} & 0 & \cdots \\ \vdots & L_Q^{(1)} & L_T^{(2)} & \hat{L}_P^{(2)} & \ddots \\ & \vdots & \ddots & \ddots & \ddots \end{array}\right]_k$$

- **We have attractivity if and only if one of these have full rank:**

$$\left[\begin{array}{ccc} L_{P,1}^{(j)} & \ldots & L_{P,M}^{(j)} \end{array}\right] \qquad \mathcal{L}_P^{(j)} := iH_P^{(j)} - \frac{1}{2}\sum_k L_{Q,k}^{(j)\dagger} L_{T,k}^{(j)}$$

- Once invariance is guaranteed, **if** there exists a generator that ensures attractivity, **then** the property is generic in the free parameters (by analytic properties of the determinant).

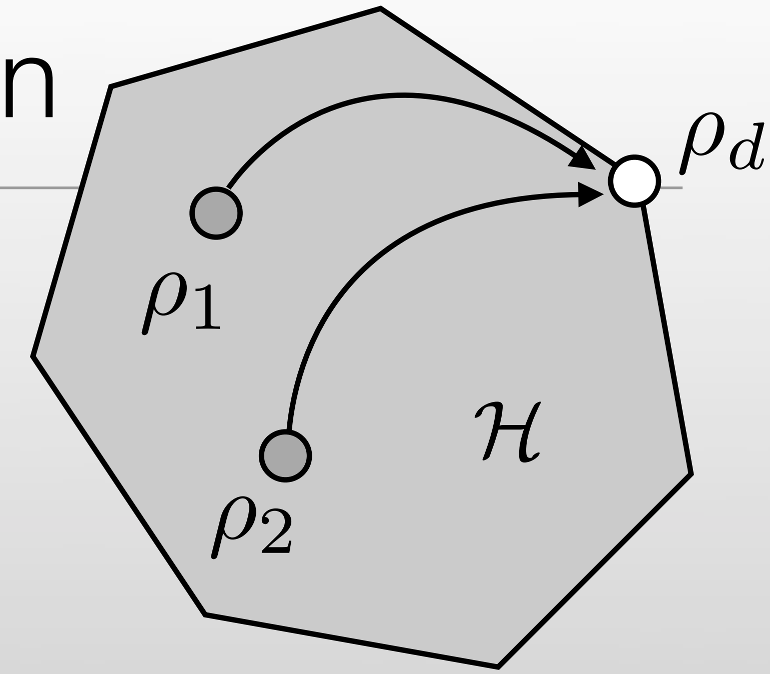
What about pre-existing Drift Dynamics?

Control Capabilities

Drift Type	Only Dissipation	Hamiltonian & Dissipation
None	DQLS Test constructive design	DQLS test QLS Randomized Test
QL Ham.	Make state invariant by **adding dissipation**, then proceed as above.	Counteract existing Hamiltonian (or add dissipation), then proceed as above.
QL Gen.	Check **if** state is invariant for existing noise; If yes, then proceed as above.	Check **if** state is invariant for existing noise; If yes, then proceed as above.

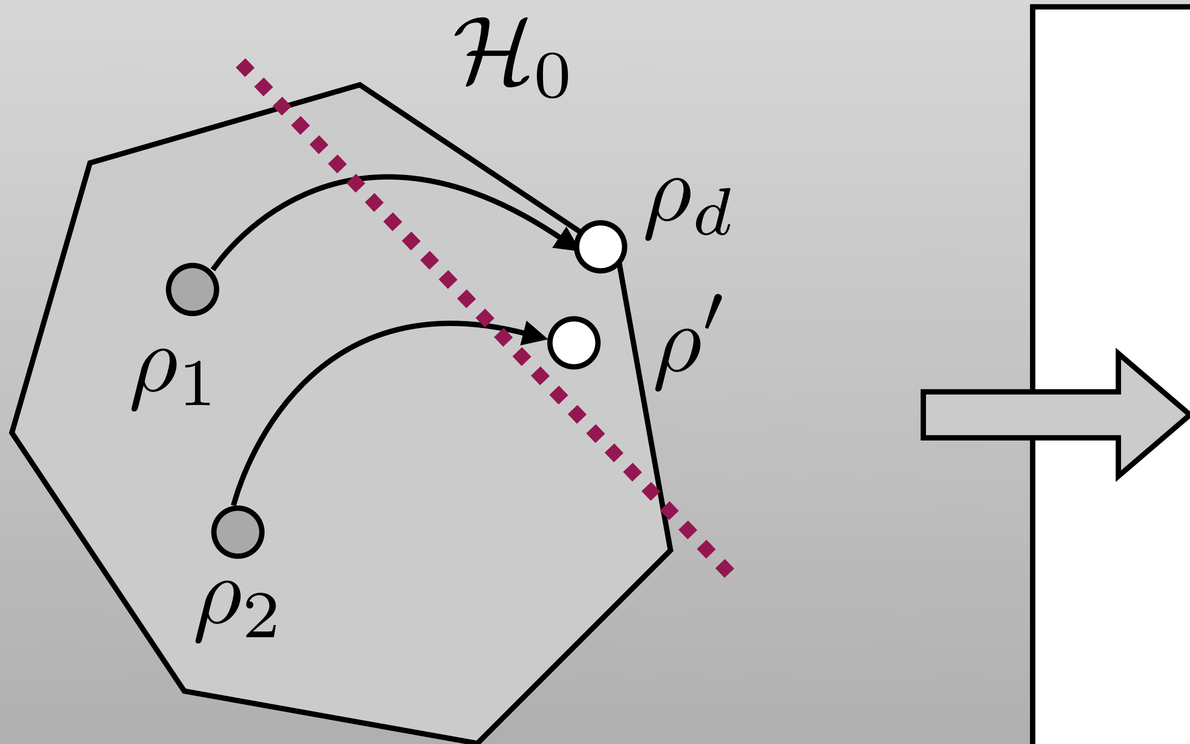
Open Challenge: Characterize class of QLS.

A Quick Look: Conditional Preparation



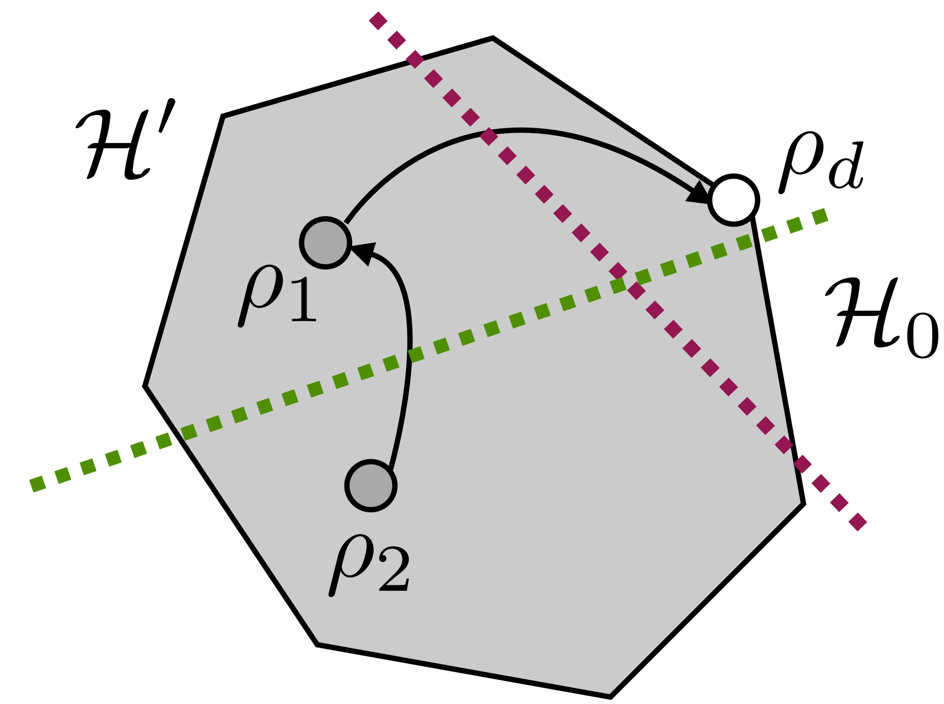
DQLS Problem: Global Stabilization Task
with Time Invariant Generators

If we relax these assumptions,
we can obtain scalable protocols
[details in the paper]



With DQLS I can only prepare

$$\mathcal{H}_0 := \bigcap_k \mathcal{H}_{\mathcal{N}_k}$$



First I prepare a subspace that
is attracted directly to ρ_d .
Problem: finding $\mathcal{H}'$!
...then design can be randomized.

Can I realize the QDS that I need?

Quantum environment engineering
by open- and closed-loop control

[T-Viola, *IEEE T. A. C., 2008;* T-Viola, *Automatica, 2009;*
T-Schirmer-Wang, *IEEE T.A.C.*, 2010;
T-et al. *IEEE T.A.C. 2011*, T-Nishio-Altafini, *IEEE T.A.C. 2013*]

Open-Loop Hamiltonian Control

- Consider $\mathcal{H} = \mathcal{H}_S \oplus \mathcal{H}_R$ and define $\mathfrak{I}_S(\mathcal{H})$ as the set of states on $\mathcal{H}_S$

- Assume (time-independent) Hamiltonian control

$$\dot{\rho}_t = -i[H + H_c, \rho_t] + \sum_{k=1}^{p} \Big(L_k \rho_t L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k,\, \rho_t\} \Big)$$

- **THEOREM:** Let $\mathfrak{I}_S(\mathcal{H})$ be invariant. Then it can be rendered attractive via Hamiltonian control **if and only if** $\mathfrak{I}_R(\mathcal{H})$ is **not** invariant, that is:

$$\exists\, k \text{ such that } L_{P,k} \neq 0$$

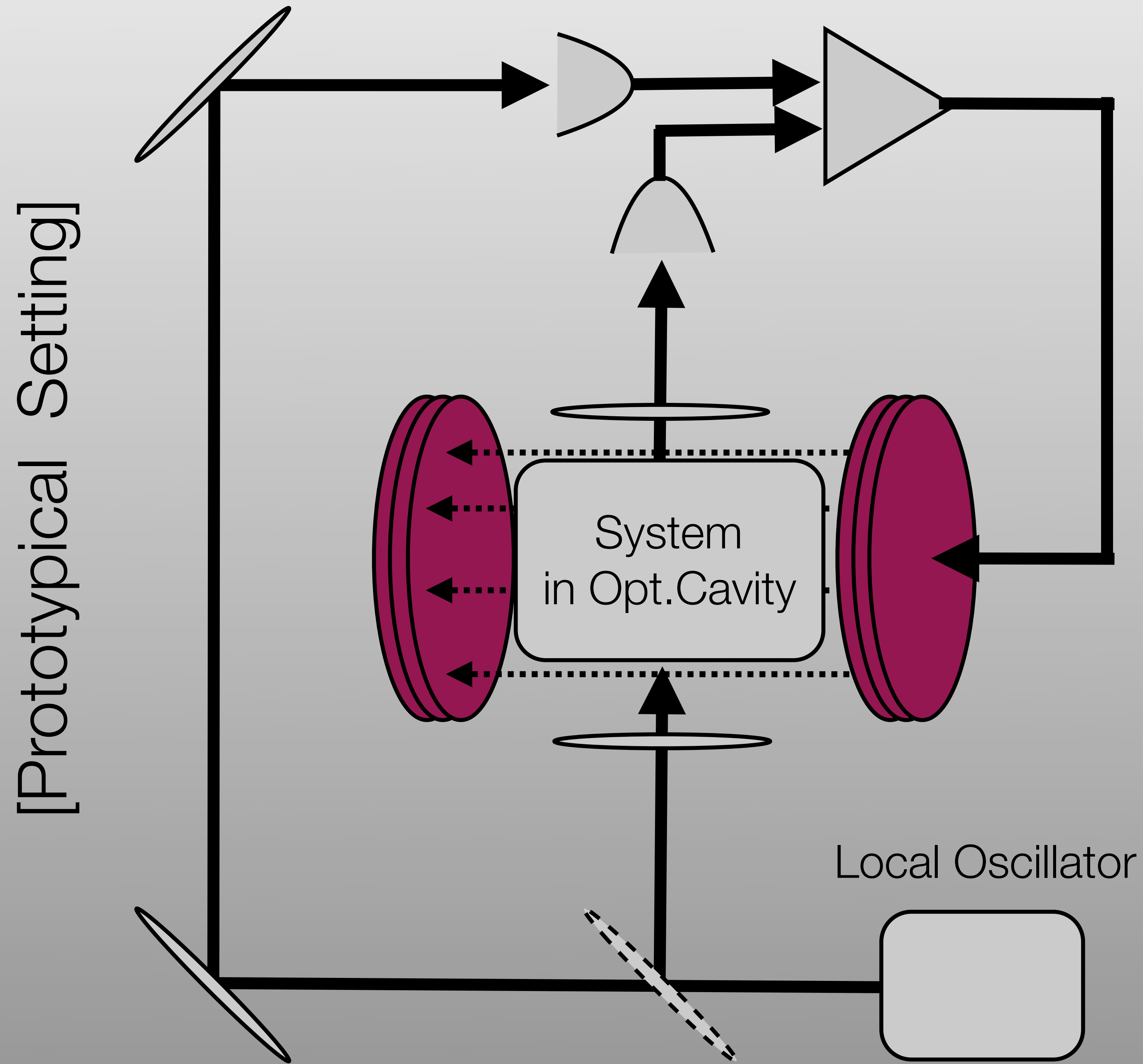
- **Good:** Constructive proof - extension with DID allows for general constraints.

Bad: $\mathfrak{I}_S(\mathcal{H})$ has to be already invariant. **How can we act on** L_k **?**

Output Feedback Scheme for Optical Systems

Environment engineering via feedback:

Homodyne detection implements indirect measurements



Output (classical) Signal:
Photocurrent

$$dy(t) = \mathrm{trace}(\mathcal{M}[\rho])dt + dW_t$$

$$\mathcal{M}[\rho] = M\rho + \rho M^\dagger$$

Assume instantaneous feedback and average over the trajectories....

Output-Feedback Control

- Hamiltonian and Feedback Control **[Wiseman-Milburn Feedback ME]:**

$$\dot{\rho}_t = -i[H + H_c, \rho_t] + L_f \rho_t L_f^\dagger - \frac{1}{2}\{L_f^\dagger L_f, \rho_t\}$$

where $L_f = M - iF$; The measurement is *fixed* , the Hamiltonian are not.

- **THEOREM:** $\mathfrak{I}_S(\mathcal{H})$ can be rendered **invariant and attractive** if and only if

$$[\Pi_S, (M + M^\dagger)] \neq 0.$$

where Π_S is the orthogonal projection on the target subspace.

- **Good:** *Constructive proof*, can generate invariant states, leaves *freedom in the choice of the controls*.

  Bad: Instantaneous feedback assumption (infinite bandwidth), single noise channel (or compatible ones), perfect detection.

Stochastic Master Equation

- **System undergoing continuous (homodyne-type) measurements:** Dynamics driven by filtering equation, aka *Stochastic Master Equation (SME)*

Conditional state:

$$d\rho_t = \big(-i[H, \rho_t] + \mathcal{D}(M, \rho_t)\big)dt + \mathcal{G}(M, \rho_t)dW_t,$$

$$\mathcal{D}(L, \rho) := L\rho L^\dagger - \frac{1}{2}(L^\dagger L\rho + \rho L^\dagger L)$$

$$\mathcal{G}(L, \rho) := \sqrt{\eta}(L\rho + \rho L^\dagger - \mathrm{Tr}((L + L^\dagger)\rho)\rho)$$

Measurement record:

$$dy_t = \sqrt{\eta}\frac{1}{2}\mathrm{Tr}(\rho_t(M + M^\dagger))dt + dW_t,$$

- **Average over trajectories:** QDS associated to the system-field interaction

$$\frac{d}{dt}\rho(t) = \mathcal{L}(\rho(t)) = -i[H, \rho(t)] + \mathcal{D}(M, \rho(t)).$$

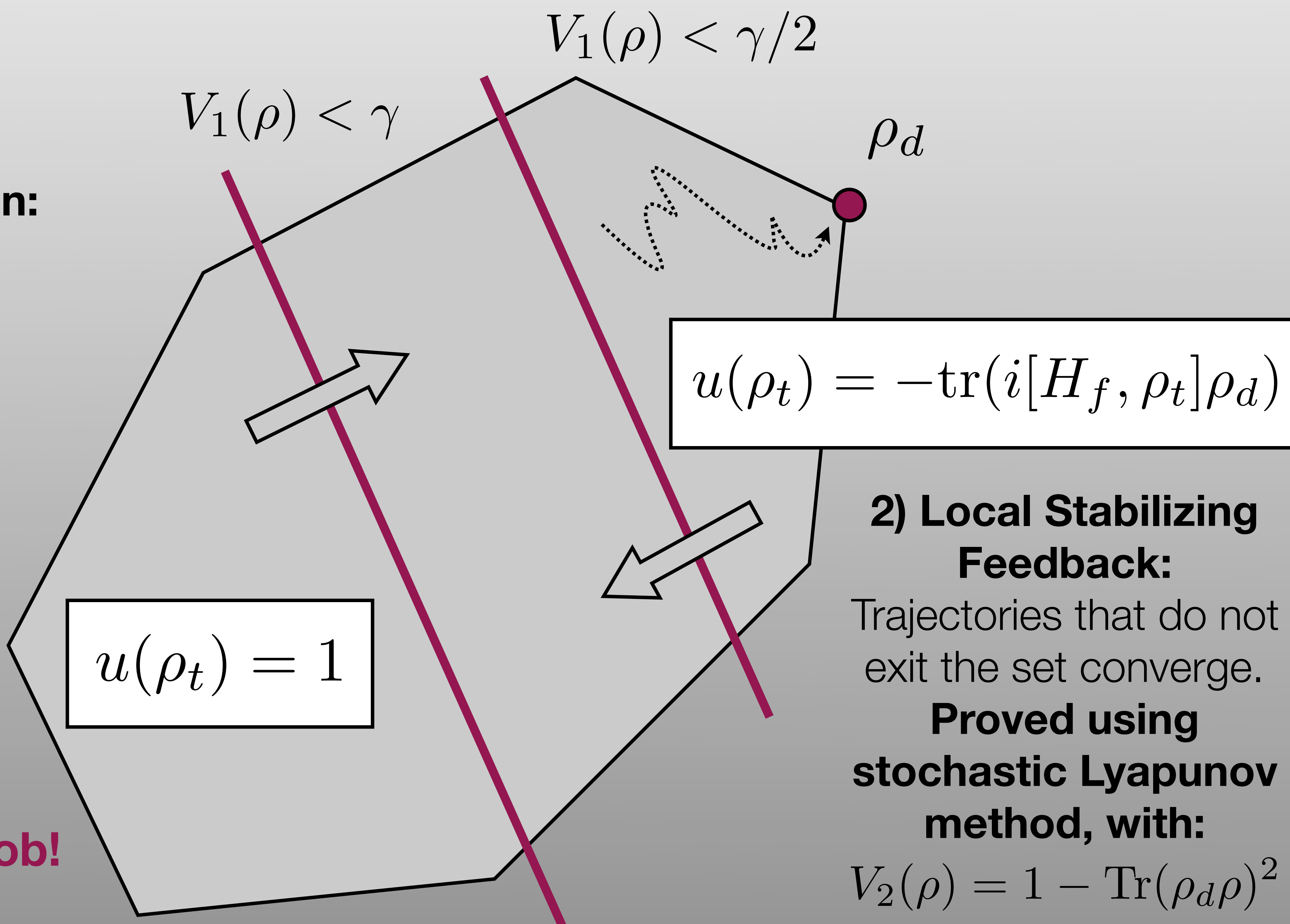
Switching Feedback Controller

Define: $V_1(\rho) = 1 - \mathrm{Tr}(\rho_d \rho)$

Exists $\gamma > 0$ such that...

Extending [Mirrahimi-van Handel SIAM Cont. Opt. 2007]

1) De-Stabilizing constant Hamiltonian:
DID-inspired design; Trajectories exit the set in finite time in expectation. Proved using Support Theorem.



2) Local Stabilizing Feedback:
Trajectories that do not exit the set converge.
Proved using stochastic Lyapunov method, with:
$V_2(\rho) = 1 - \mathrm{Tr}(\rho_d \rho)^2$

Open loop control does most of the job!

The Message...

- Using system-theoretic methods, we derived a framework for
  - **Stability analysis of QDS on multipartite systems;**
  - **Tests for checking DQLS and QLS states;**
  - **Constructive results for pure entangled state preparation under locality constraints;**
  - **Scalable protocols for conditional preparation;**
  - **Stabilizability analysis for different control models;**

➡**Open problems:**
Better characterization of QLS states;
Speed of convergence (when the system size grows - scalability);
Robustness;

➡**Next:**
Mixed states; Phase transitions;
Discrete-time models; Non-Markovian Models; ...

Overview: From Theory towards Applications