Scattering induced current in a tight binding band

Joint work with

S. De Bièvre (Lille) and L. Bruneau (Cergy)

Cergy — May 2011

1. Repeated interactions

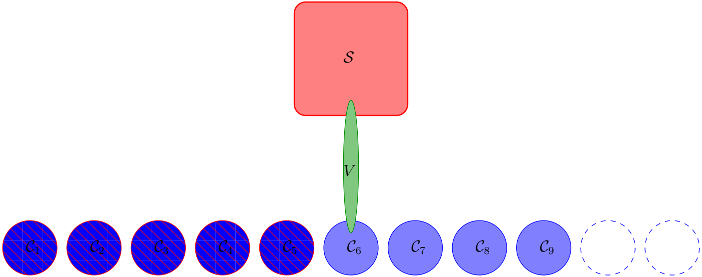
2. The model

3. Results

4. Markovian dynamics

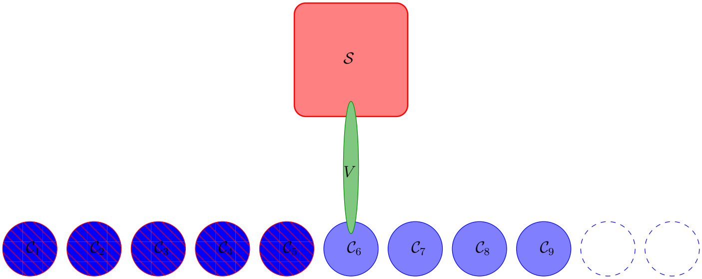
Repeated interactions

- Markovian quantum dynamics



Repeated interactions

- Markovian quantum dynamics
- Kümmerer-Maassen (2000)
- Attal-Pautrat (2006), Attal-Joye (2007)
- Bruneau-Joye-Merkli (2006),(2007),(2008),(2010)
- Bruneau-Pillet (2009)
- ...



Accelerated Bloch electron

$$H_{\mathrm{p}} = -\Delta + FX \quad \big(\text{on } \mathcal{H}_{\mathrm{p}} = \ell^2(\mathbb{Z}) \qquad\qquad \big)$$

$$-\Delta = \sum_{x\in\mathbb{Z}} \big[2|x\rangle\langle x| - |x\rangle\langle x+1| - |x+1\rangle\langle x|\big], \qquad X = \sum_{x\in\mathbb{Z}} x\,|x\rangle\langle x|$$

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Wannier-Stark Ladder: $H_{\mathrm{p}}\psi_k = E_k\psi_k$

$$E_k = 2 - Fk, \qquad \psi_k(x) = J_{k-x}\left(\frac{2}{F}\right), \quad (k\in\mathbb{Z})$$

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Bloch oscillations: $X(t) = \mathrm{e}^{\mathrm{i}tH_{\mathrm{p}}} X \mathrm{e}^{-\mathrm{i}tH_{\mathrm{p}}} = X + \frac{4}{F}\sin\left(\frac{Ft}{2}\right)\sin\left(\xi + \frac{Ft}{2}\right)$

Accelerated Bloch electron + 2 level atom

$$\mathcal{H}_{\mathrm{a}} = \Gamma_-(\mathbb{C}) \simeq \mathbb{C}^2$$

$$H_{\mathrm{a}} = E b^* b, \qquad \{b^*, b\} = 1, \qquad b^2 = 0$$

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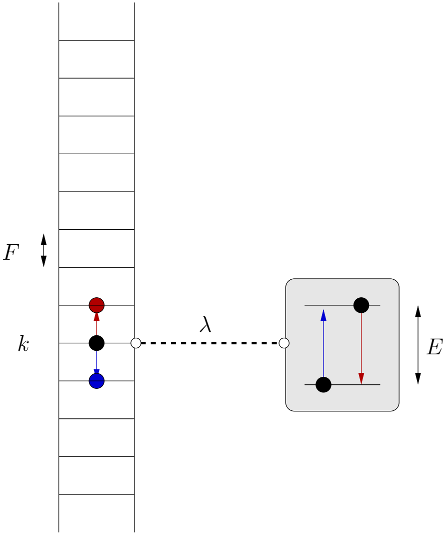
$$V = \sum_{x\in\mathbb{Z}} \left[|x+1\rangle\langle x| \otimes b^* + |x\rangle\langle x+1| \otimes b \right] = Tb^* + T^*b$$

$$H = H_{\mathrm{p}} + H_{\mathrm{a}} + \lambda V = 2 - T(1-\lambda b^*) - T^*(1-\lambda b) - FX + Eb^*b$$

Accelerated Bloch electron + 2 level atom



Accelerated Bloch electron + 2 level atom

$$
\begin{aligned}
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Bloch + Rabi oscillations

Rabi frequency: $\omega_0 = \sqrt{(E-F)^2 + 4\lambda^2}$

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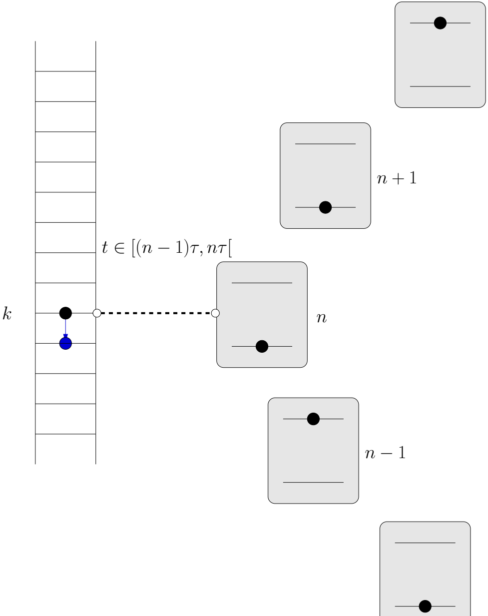
In this presentation we consider only the resonant case $F = E$

Repeated electron-atom interactions



Repeated electron-atom interactions

Piecewise constant time-dependent Hamiltonian

$$H(t) = H_n = H_{\mathrm{p}} + \sum_j H_{\mathrm{a},j} + \lambda V_n \qquad (t \in [(n-1)\tau, n\tau[)$$

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Initial state: $\rho_0 = \rho_{\mathrm{p}} \otimes \rho_{\mathrm{a}}^{\otimes\infty}$, $\qquad \rho_{\mathrm{a}}^{\otimes\infty} = \bigotimes_j \frac{\mathrm{e}^{-\beta H_{\mathrm{a},j}}}{\mathrm{tr}\left(\mathrm{e}^{-\beta H_{\mathrm{a},j}}\right)}$

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Electronic state after n interactions: $\rho_{\mathrm{p},n} = \mathrm{tr}_{\mathcal{H}_\mathrm{a}^{\otimes\infty}}\left(U^{(n)} \rho_0 U^{(n)*}\right)$

Position "Full Counting Statistics"

- At time $t = 0^-$ measure the position of the particle

$$\longrightarrow \qquad X_0 \in \mathbb{Z} \text{ with probability } \langle X_0 | \rho_{\rm p} X_0 \rangle$$

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- What is the statistics of $\Delta X_n = X_n - X_0$?

Main results

Assume: $\lambda \neq 0, \tau\omega_0 \notin 2\pi\mathbb{Z} \Rightarrow p = \sin^2\left(\frac{\omega_0\tau}{2}\right) \in]0,1],$

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Drift and diffusion constant:

$$\lim_{n\to\infty}\left\langle \frac{\Delta X_n}{n\tau}\right\rangle = v_d, \qquad \lim_{n\to\infty}\left\langle \frac{(\Delta X_n - v_d n\tau)^2}{2n\tau}\right\rangle = D$$

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Einstein's relation:

$$\mu = \lim_{E\to 0}\frac{v_d}{E} = \frac{\beta p}{2\tau} = \lim_{E\to 0}\beta D$$

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Central Limit Theorem:

$$\lim_{n\to\infty}\left\langle f\left(\frac{\Delta X_n - v_d n\tau}{\sqrt{2Dn\tau}}\right)\right\rangle = \int f(x)\mathrm{e}^{-x^2/2}\frac{\mathrm{d}x}{\sqrt{2\pi}}$$

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$$\theta(\alpha) = (1-p) + p\frac{\cosh\left(\left(\frac{1}{2}-\alpha\right)\beta E\right)}{\cosh\left(\frac{1}{2}\beta E\right)}$$

Cumulant generating function:

$$g(\eta) = \lim_{n\to\infty}\frac{1}{n}\log\left\langle e^{\eta\Delta X_n}\right\rangle = \log\theta\left(-\frac{\eta}{\beta E}\right)$$

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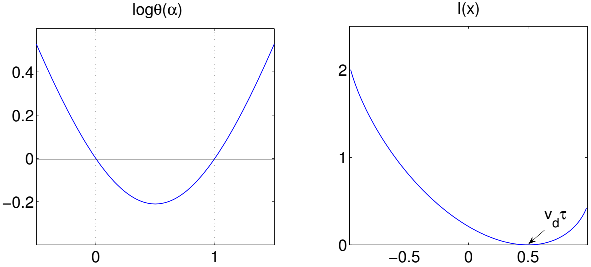
Large Deviation Principle:

$$\lim_{n\to\infty}\frac{1}{n}\log \text{Prob}\left[\frac{\Delta X_n}{n} \in J\right] = -\inf_{x\in J} I(x), \quad I(x) = \sup_{\eta\in\mathbb{R}}\left[\eta x - g(\eta)\right]$$

Main results



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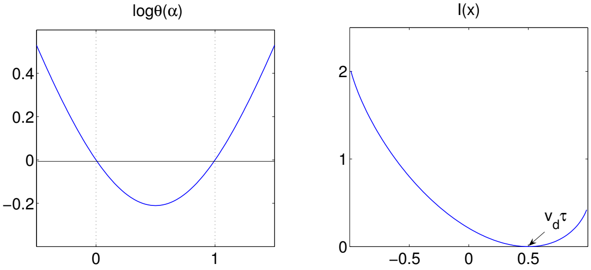
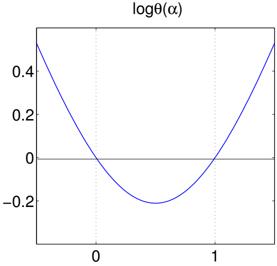
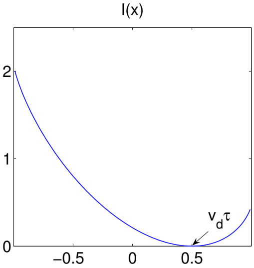


Fluctuation Relation:

$$\theta(1-\alpha) = \theta(\alpha) \Rightarrow I(-v) = I(v) + \beta E v$$

Main results





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Fluctuation Theorem:

$$\frac{\mathrm{Prob}\left[\frac{\Delta X_n}{n\tau} = -v\right]}{\mathrm{Prob}\left[\frac{\Delta X_n}{n\tau} = v\right]} \simeq e^{n\beta E v}$$

Effective evolution

$$\rho_{\mathrm{p},n} = \mathrm{tr}_{\mathcal{H}_{\mathrm{a}}^{\otimes\infty}}\left(U^{(n)}(\rho_{\mathrm{p}} \otimes \rho_{\mathrm{a}}^{\otimes\infty})U^{(n)*}\right)$$

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$$\rho_{\mathrm{p},n} = \mathrm{tr}_{\mathcal{H}_\mathrm{a}^{\otimes\infty}}\left(U^{(n)}(\rho_\mathrm{p}\otimes\rho_\mathrm{a}^{\otimes\infty})U^{(n)*}\right) = \mathcal{L}^n(\rho_\mathrm{p}),$$

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More generally:

$$\mathrm{tr}_{\mathcal{H}_{\mathrm{a}}^{\otimes\infty}} \left((I \otimes (\rho_{\mathrm{a}}^{\otimes\infty})^{\alpha}) U^{(n)} (\rho \otimes (\rho_{\mathrm{a}}^{\otimes\infty})^{1-\alpha}) U^{(n)*} \right) = \mathcal{L}_{\alpha}^{n}(\rho)$$

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Remark. $\mathcal{L}_\alpha$ is a CP map !

Symmetries

Gauge Invariance: $N = \dfrac{H_\mathrm{p} - 2}{E} + \dfrac{H_\mathrm{a}}{E}$

$$[H, N] = 0 \implies \mathcal{L}_\alpha\left(\mathrm{e}^{\mathrm{i}\tau H_\mathrm{p}} A\, \mathrm{e}^{-\mathrm{i}\tau H_\mathrm{p}}\right) = \mathrm{e}^{\mathrm{i}\tau H_\mathrm{p}} \mathcal{L}_\alpha(A) \mathrm{e}^{-\mathrm{i}\tau H_\mathrm{p}} =: \widetilde{\mathcal{L}}_\alpha(A)$$

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$$\mathcal{C}(A) = C_\mathrm{p} A C_\mathrm{p}^* \implies \mathcal{L}_\alpha \circ \mathcal{C} = \mathcal{C} \circ \mathcal{L}_{1-\alpha}^\dagger$$

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spectral radius$(\mathcal{L}_\alpha)$ = spectral radius$(\mathcal{L}_{1-\alpha})$

Kraus representation

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2. $\widetilde{\mathcal{L}}_\alpha$ is a CP map. By known results [Schrader 2001]

$$\text{spectral radius}(\widetilde{\mathcal{L}}_\alpha) = \theta(\alpha)$$

The symmetry $\theta(1 - \alpha) = \theta(\alpha)$ is a consequence of TRI

The cumulant generating function

$$g_n(\eta) = \log \left\langle e^{\eta \Delta X_n} \right\rangle = \log \operatorname{tr} \left(\mathcal{L}_0^n \left(\rho_{\mathrm{p}} e^{-\eta X} \right) e^{\eta X} \right)$$

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$$\begin{aligned} \frac{1}{n} g_n(\eta) &\simeq \frac{1}{n} \log \operatorname{tr} \left(\widetilde{\mathcal{L}}^n_{-\eta/\beta E} \left(\rho_p \right) I \right) \\ &= \frac{1}{n} \log \operatorname{tr} \left(\rho_p \widetilde{\mathcal{L}}^{\dagger n}_{-\eta/\beta E} \left(I \right) \right) \\ &= \log \theta(-\eta/\beta E) \end{aligned}$$

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- Its real interest is to display a new type of "Fluctuation Theorem": neither *transient* (does not hold for finite time) nor *stationary* (there is no NESS).
- The non-resonant case $E \neq F$ is completely similar with one important difference: Einstein's relation holds in the sense

$$\mu = \lim_{E \to 0} \frac{v_d}{E} = \lim_{E \to 0} \beta D,$$

i.e., the driving force is E (the atomic Bohr frequency) not F (the electric field strength).

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- We still miss a general theory of entropic fluctuations for RI systems. In this perspective, the present work is just a very simple case study.
- It shows however that general ideas, e.g.

$$\text{TRI} \;\Rightarrow\; \text{Fluctuation theorems}$$

can be adapted to this setting.