Open Quantum Systems
Out of Equilibrium

Recent Developments in
Nonequilibrium Quantum Statistical Mechanics

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Université du Sud – Toulon-Var

Overview

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- Outlook: Towards a mathematical theory of quantum fluctuations in NESS.

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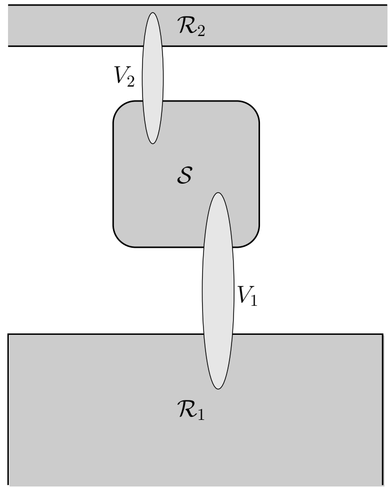
- Araki stability: (τ, β)-KMS $\omega_\beta \longleftrightarrow (\tau_V, \beta)$-KMS $\omega_\beta^V \sim \omega_\beta$ (mutually normal).

Open Quantum Systems



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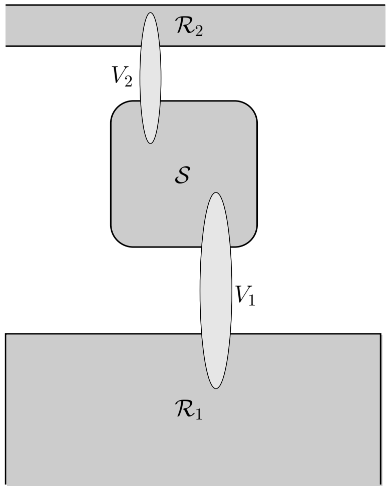

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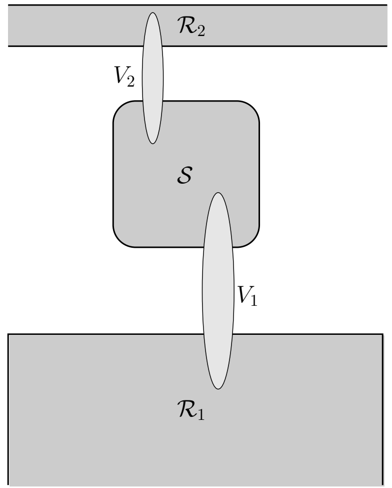

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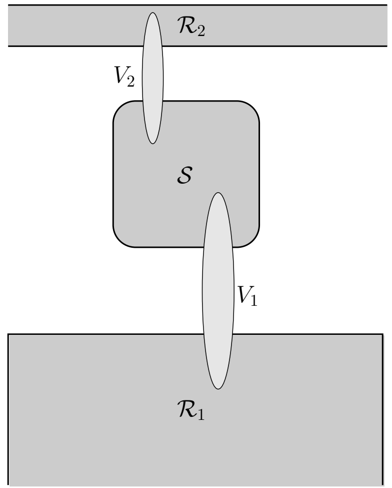

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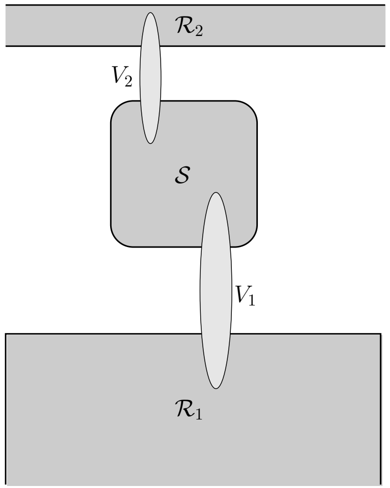

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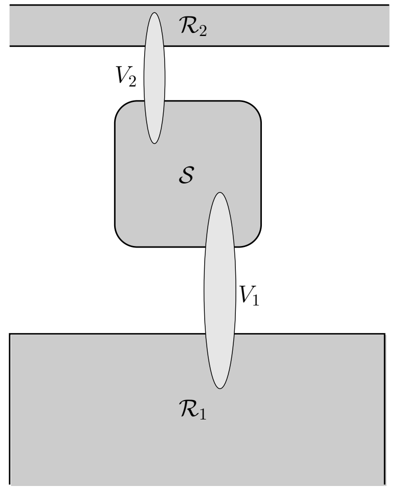

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[Ruelle '00]

For all $A \in \mathcal{O}$ and all $\omega_{\vec{\beta}}$–normal state η

$$\lim_{t\to\infty} \eta \circ \tau_V^t(A) = \omega_{\vec{\beta}}^{+}(A).$$

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Hilbert space approach ?

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Basic Problem 1. Existence of NESS.

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Φ_j = heat flux out of $\mathcal{R}_j$ = $-$rate of change of energy in $\mathcal{R}_j$.

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Theorem. [Ruelle ’01], [Jakšić-P ’01] If η is $\omega_{\vec{\beta}}$-normal then

$$\mathrm{Ep}(\omega_{\vec{\beta}}^{\,+}) = - \lim_{t \to \infty} \frac{\mathrm{Ent}(\eta \circ \tau_V^t | \omega_{\vec{\beta}})}{t} \geq 0.$$

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Basic Problem 2. Strict positivity of entropy production

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Theorem. [Jakšić-P '02] If $\mathrm{Ep}(\omega_{\vec{\beta}}^{\,+}) > 0$ then $\omega_{\vec{\beta}}^{\,+}$ is not $\omega_{\vec{\beta}}$-normal. Reciproquely, if $\omega_{\vec{\beta}}^{\,+}$ is not $\omega_{\vec{\beta}}$-normal and

$$\limsup_{t\to\infty} \left| \int_0^t \left(\omega_{\vec{\beta}} \circ \tau_V^s(\sigma) - \omega_{\vec{\beta}}^{\,+}(\sigma) \right) ds \right| < \infty,$$

then $\mathrm{Ep}(\omega_{\vec{\beta}}^{\,+}) > 0$.

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Theorem. [Aschbacher-Jakšić-Pautrat-P '06] If the C^*-dynamical system $(\mathcal{O}^+, \tau|_{\mathcal{O}^+}, \omega|_{\mathcal{O}^+})$ is mixing then

$$\lim_{t\to\infty} \eta \circ \tau_V^t(A) = \omega^+(A),$$

for all $A \in \mathcal{O}$ and all ω-normal states η.

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1. γ^+ exists.
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Strategy: Cook's method, $\lim_{t\to\infty} \tau_V^t \circ \tau^t|\otimes_{j>0}\mathcal{O}_j$, Araki-Dyson expansion

$$\tau_V^t(A) = \tau^t(A) + \sum_{n>0} \int_{0<t_n<\cdots<t_1<t} i[\tau^{t_n}(V), i[\cdots, i[\tau^{t_1}(V), \tau^t(A)]\cdots]]\, dt_1 \cdots dt_n.$$

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Strategy: Gelfand triplet $\mathcal{K} \subset \mathcal{H}_\omega \subset \mathcal{K}'$, dense subalgebra $\tilde{\mathcal{O}} \subset \mathcal{O}$ s.t. $\pi_\omega(\tilde{\mathcal{O}})\Omega_\omega \subset \mathcal{K}$.
if $\exists \Psi \in \mathcal{K}'$ s.t. $\mathrm{w}^* - \lim_{t\to\infty} \mathrm{e}^{-iLt}\phi = (\Omega_\omega, \phi)\Psi$ for all $\phi \in \mathcal{K}$, $\quad (*)$
$\Downarrow$
$\lim_{t\to\infty} \eta \circ \tau_V^t(A) = \omega_{\vec{\beta}}{}^+(A)$ for all $A \in \mathcal{O}$ and all $\omega_{\vec{\beta}}$-normal state η,
$\omega_{\vec{\beta}}{}^+(A) = (\Omega_\omega, \pi_\omega(A)\Psi)$ for $A \in \tilde{\mathcal{O}}$.

Construction of NESS: The Liouvillean Approach

Work in the GNS representation: [Jakšić-P '02], [Merkli-Mueck-Sigal '06].

Assume that the reference state ω is τ-invariant and modular (e.g. ω is multi-KMS).
Tomita-Takesaki modular theory
$\Downarrow$
Dynamics in GNS representation $(\mathcal{H}_\omega, \pi_\omega, \Omega_\omega)$:
$\pi_\omega(\tau_V^t(A)) = \mathrm{e}^{iLt}\pi_\omega(A)\mathrm{e}^{-iLt}$ with $L^*\Omega_\omega = 0$,
uniquely defines the C-Liouvillean L (non-selfadjoint!).

Strategy: Gelfand triplet $\mathcal{K} \subset \mathcal{H}_\omega \subset \mathcal{K}'$, dense subalgebra $\tilde{\mathcal{O}} \subset \mathcal{O}$ s.t. $\pi_\omega(\tilde{\mathcal{O}})\Omega_\omega \subset \mathcal{K}$.
if $\exists \Psi \in \mathcal{K}'$ s.t. $\mathrm{w}^* - \lim_{t\to\infty} \mathrm{e}^{-iLt}\phi = (\Omega_\omega, \phi)\Psi$ for all $\phi \in \mathcal{K}$, $\quad (*)$
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Under sufficient regularity assumptions on V spectral deformation techniques allow to control resonances of L and prove $(*)$.

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$$\overrightarrow{\beta}_{\mathrm{equ}} = (\beta, \beta, \ldots, \beta).$$

$$\omega_{\overrightarrow{\beta}}^{\;+} = \lim_{t\to\infty} \omega_{\overrightarrow{\beta}} \circ \tau_V^t, \quad (|\overrightarrow{\beta} - \overrightarrow{\beta}_{\mathrm{equ}}| < \epsilon).$$

Transport coefficients (Onsager matrix):

$$L_{kj} = -\partial_{\beta_j} \omega_{\overrightarrow{\beta}}^{\;+}(\Phi_k)|_{\overrightarrow{\beta} = \overrightarrow{\beta}_{\mathrm{equ}}}.$$

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Remark. Order of limits: 1st thermodynamic limit, 2nd long time limit, 3rd weak forcing limit.

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Quasi-free state $\mu_L(W(A)) = \mathrm{e}^{-L(A,A)/2}$

$\Downarrow$

regular GNS representation $(\mathcal{H}_L, \pi_L, \Omega_L)$ with $\pi_L(W(A)) = \mathrm{e}^{i\phi_L(A)}$.

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Basic Problem 4. Simple Quantum Dynamical CLT:

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Theorem. [Jakšić-Pautrat-P '07] If QD-CLT holds for $\mathfrak{C}$ then

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holds for all bounded Borel functions $f_1, \ldots, f_n$.

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- Quasifree fermions/bosons and XY spin chains ($\rightarrow$ independent particles models): C^*- scattering reduces to Hilbert space scattering leading to Landauer-Büttiker formalism,
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Assumptions. For all j's:

(A1) $\mathfrak{h}_j = L^2(\mathbb{R}_+, ds) \otimes \mathfrak{K}_j$ and $h_j = s$ (spectral representation).

(A2) For some $\delta > 0$ and all a: $e^{-as}\alpha_j(|s|) \in H^2(\{|\mathrm{Im}s| < \delta\}) \otimes \mathfrak{K}_j$ (analyticity).

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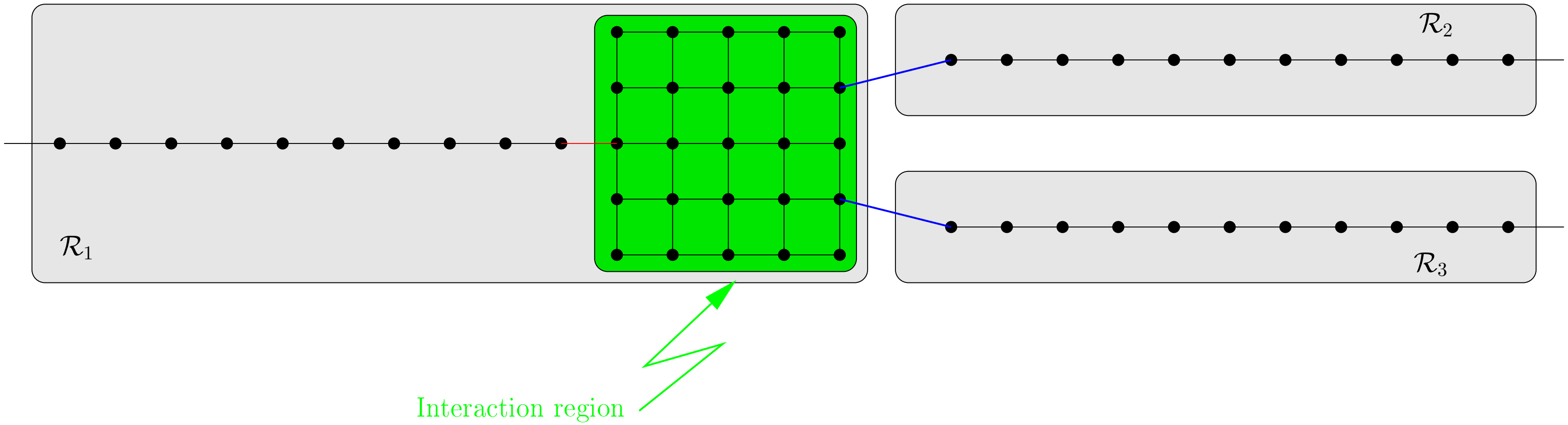
Theorem. [Jakšić-P '02], [Jakšić-Ogata-P, '06] Assume (A1)-(A3) and let $0 < \gamma_1 < \gamma_2$ be given. Then there exists $\Lambda > 0$ such that, for all $0 < |\lambda| < \Lambda$ and $\gamma_1 < \beta_j < \gamma_2$:

1. There exists a NESS $\omega_{\vec{\beta}}^{+}$.
2. If the β_j's are not all equal then $\omega_{\vec{\beta}}^{+}$ is not $\omega_{\vec{\beta}}$-normal and $\mathrm{Ep}(\omega_{\vec{\beta}}^{+}) > 0$.
3. The Green-Einstein-Kubo formulas (1) and (2) hold as well as the Onsager reciprocity relations.

Example 2: Locally Interacting Fermi Gases



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$$V = \lambda \sum_{k=1}^{K} \prod_{j=1}^{n_k} a^*(u_{jk}) a(v_{jk}).$$

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$\tau_j^t(a_j(f)) = a_j(\mathrm{e}^{ith_j} f)$ Bogoliubov automorphism.

Gauge-invariant coupling/interaction

$$V = \lambda \sum_{k=1}^{K} \prod_{j=1}^{n_k} a^*(u_{jk}) a(v_{jk}).$$

Assumptions. Set $\mathfrak{h} \equiv \oplus_j \mathfrak{h}_j$, $h \equiv \oplus_j h_j$ and $\mathcal{D}_0 \equiv \{u_{jk}, v_{jk}\}$:

(A1) There is a dense subspace $\mathcal{D} \subset \mathfrak{h}$ containing $\mathcal{D}_0$ and such that

$$\int_{-\infty}^{\infty} |(f, e^{ith} g)| \, dt < \infty,$$

for all $f, g \in \mathcal{D}$.

(A2) $h\mathcal{D}_0 \subset \mathcal{D}$.

(A3) There is a complex conjugation on $\mathfrak{h}$ which commute with h and such that u_{jk}, v_{jk} are real (Time reversal invariance).

Example 2: Locally Interacting Fermi Gases

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Theorem. [Jakšić-Ogata-P '06], [Jakšić-Pautrat-P '07] If (B1) holds then there exists $\Lambda > 0$ such that, for $0 < |\lambda| < \Lambda$:

1. There exists a NESS $\omega_{\vec{\beta}}^{+}$.
2. If (B2) also holds then the Green-Einstein-Kubo formula (1) holds.
3. If, in addition, (B3) holds then the Green-Einstein-Kubo formula (2) and the Onsager reciprocity relations hold. Moreover QDCLT holds for $\mathfrak{C} = \{\Phi_1, \Phi_2, \ldots\}$.

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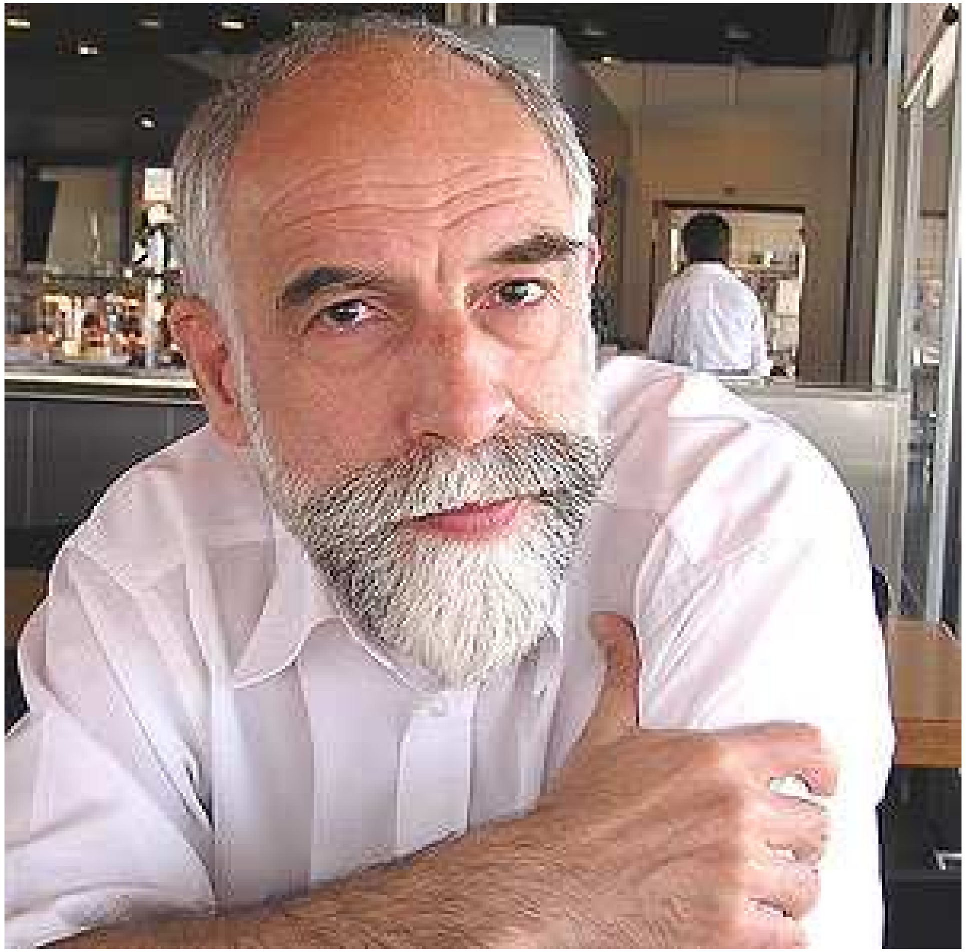
- Scattering approach to repeated interactions [Kümmere-Maassen ’00].

Merci Jürg pour tout ce que tu nous as apporté



et ...

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