Landauer's Principle in Quantum Statistical Mechanics

Joint work with Vojkan Jakšić (McGill University)

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1. Introduction – A thermodynamic argument
2. Landauer's Principle in statistical mechanics
3. Algebraic framework – Abstract Landauer's Principle
4. Tightness of Landauer's bound
5. Conclusion

1. Introduction

Taming Maxwell's demon: a never ending story made short

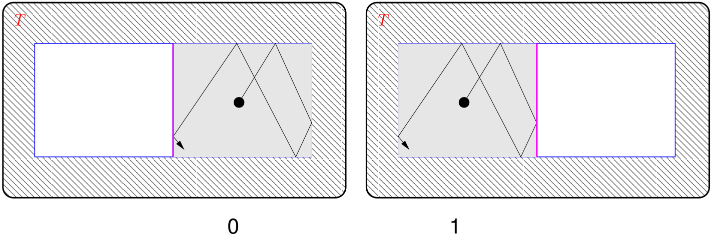
- 1871: Maxwell's demon violates the 2^{nd} Law
- 1929: Szilard's engine converts information into work
- 1956: Brillouin: irreversibility of quantum measurement processes
- 1961: Landauer: *logically irreversible* operations dissipate heat

$$\Delta Q = k_{\mathrm{B}} T \log 2 \text{ per bit}$$

- 1982: Bennett exorcises the demon
- 1999: Earman-Norton criticism...
- ... many attempts to "prove" Landauer's principle from "first principles" (stat. mech.) or conceive classical and quantum systems that violate it ...

Thermodynamic “derivation” of Landauer’s Principle

The ideal gas 1-bit memory ($pV = k_\mathrm{B} T$)



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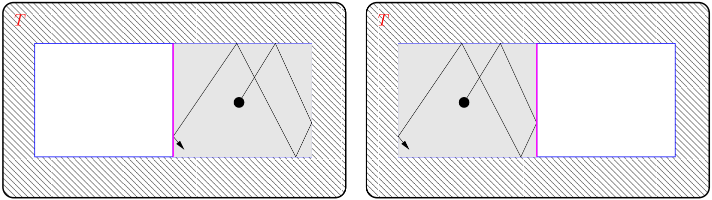


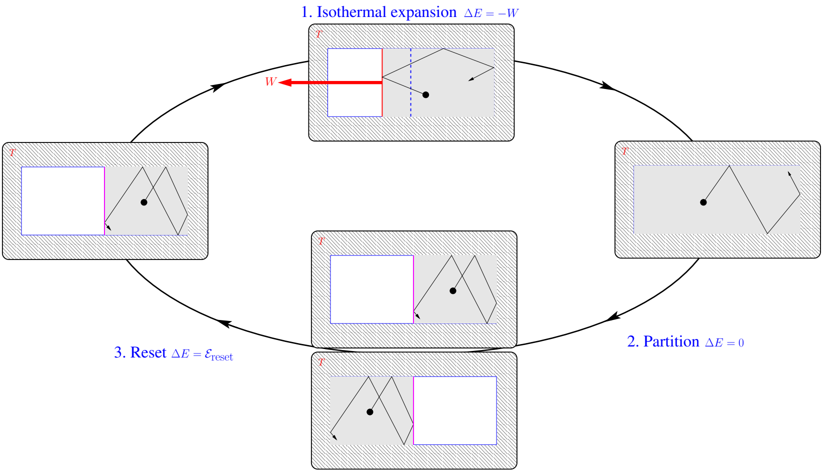
0 1

Assume there is a process which perform the reset operation **(0 or 1)** $\to 0$ with energy cost

$$\mathcal{E}_{\text{reset}}$$

Thermodynamic “derivation” of Landauer’s Principle

Build a cyclic process



Thermodynamic “derivation” of Landauer’s Principle

Work extracted during isothermal quasi-static expansion

$$W = \int_{V/2}^{V} p\, dV = \int_{V/2}^{V} \frac{k_{\mathrm{B}} T}{V}\, \mathrm{d}V = k_{\mathrm{B}} T \log 2$$

The second law imposes

$$\mathcal{E}_{\mathrm{reset}} \geq k_{\mathrm{B}} T \log 2$$

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[Landauer '61] The energy injected in the reset process is released as heat in the reservoir. $k_{\mathrm{B}} T \log 2$ is the minimal energy dissipated by a reset operation. Moreover

$$k_{\mathrm{B}} T \log 2 = T \Delta S$$

ΔS being the decrease in entropy of the system in the reseting process (erasing entropy). Note that Landauer's bound $\mathcal{E}_{\mathrm{reset}} \geq T \Delta S$ is saturated by the reverse process of quasi-static isothermal compression.

2. Landauer's Principle from statistical mechanics

[Earman-Norton 1999, Bennett 2003, Leff-Rex 2003, ...] All known derivations of Landauer's Principle assume the validity of one or another form of the 2^{nd} Law.

[Shizume 1995, Piechocinska 2000, ...] Landauer's Principle from classical and quantum microscopic dynamics of specific systems

[Reeb-Wolf 2014] *Much of the misunderstanding and controversy around Landauer's Principle appears to be due to the fact that its general statement has not been written down formally or proved in a rigorous way in the framework of quantum statistical physics*

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[Reeb-Wolf 2014] *Much of the misunderstanding and controversy around Landauer's Principle appears to be due to the fact that its general statement has not been written down formally or proved in a rigorous way in the framework of quantum statistical physics*

This formulation will definitively not close the philosophical discussions about Maxwell's demon and the relation between thermodynamics and information theory, but at least it provides a sound statement with well defined assumptions.

Landauer's Principle in statistical mechanics [Reeb-Wolf '14]

Finite quantum system $\mathcal{S}$ coupled to finite reservoir $\mathcal{R}$ at temperature $T > 0$

- Finite dimensional Hilbert space $\mathcal{H} = \mathcal{H}_\mathcal{S} \otimes \mathcal{H}_\mathcal{R}$, reservoir Hamiltonian $H_\mathcal{R}$
- Product initial state + thermal reservoir $\omega_i = \rho_i \otimes \nu_i$

$$\nu_i = \mathrm{e}^{-(\beta H_\mathcal{R} + \log Z)}, \qquad \beta = \frac{1}{k_\mathrm{B} T}, \qquad Z = \mathrm{tr}\left(\mathrm{e}^{-\beta H_\mathcal{R}}\right)$$

- Unitary state transformation $U : \omega_i \mapsto \omega_f = U\omega_i U^*$
- Reduced final states

$$\rho_f = \mathrm{tr}_{\mathcal{H}_\mathcal{R}}(\omega_f), \qquad \nu_f = \mathrm{tr}_{\mathcal{H}_\mathcal{S}}(\omega_f)$$

- Energy dissipated in the reservoir $\mathcal{R}$:

$$\Delta Q = \mathrm{tr}((\nu_f - \nu_i) H_\mathcal{R})$$

- Decrease in entropy of the system $\mathcal{S}$:

$$\Delta S = S(\rho_i) - S(\rho_f)$$

where $S(\rho) = -k_\mathrm{B}\,\mathrm{tr}\,(\rho \log \rho)$ is the von Neumann entropy of ρ

Landauer's Principle in statistical mechanics

Landauer's bound [Reeb-Wolf '14, Tasaki '00]

$$\Delta Q = T(\Delta S + \sigma), \qquad \sigma \geq 0 \tag{1}$$

$\sigma = 0$ iff $\Delta Q = T\Delta S = 0$, in which case one has

$$\nu_f = \nu_i$$

and ρ_f is unitarily equivalent to ρ_i.

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Remark 1. If $\mathcal{S}$ is a qubit,

$$\rho_i = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}, \qquad \rho_f = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

then the transformation $\rho_i \to \rho_f$ implements the state change **(**0 **or** 1**)** $\to$ 0 and

$$T\Delta S = k_{\mathrm{B}} T \log 2$$

However, this transition can not be induced by a finite reservoir at positive temperature (more later).

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Remark 2. Von Neumann entropy is the quantum version of Shannon information theoretic entropy. It only coincides with thermodynamic (Clausius) entropy for thermal equilibrium states.

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Proof. Very basic tool: Relative entropy of two states ω, ν, given by

$$S(\omega|\nu) = \begin{cases} \mathrm{tr}(\omega(\log\omega - \log\nu)) & \text{if } \mathrm{Ran}(\omega) \subset \mathrm{Ran}(\nu); \\ +\infty & \text{otherwise}; \end{cases}$$

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set $k_{\mathrm{B}} = 1$

$$0 \leq \sigma = S(\omega_f|\rho_f \otimes \nu_i)$$

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$$\log\rho_f \otimes \nu_i = \log\rho_f \otimes I + I \otimes \log\nu_i$$

$$0 \leq \sigma = S(\omega_f|\rho_f \otimes \nu_i) = \mathrm{tr}(\omega_f \log\omega_f) - \mathrm{tr}(\omega_f(log\rho_f \otimes I)) - \mathrm{tr}(\omega_f(I \otimes \log\nu_i))$$

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$$\omega_f \log\omega_f = U(\omega_i \log\omega_i)U^*$$

$$\begin{aligned} 0 \leq \sigma = S(\omega_f|\rho_f \otimes \nu_i) &= \mathrm{tr}(\omega_f \log\omega_f) - \mathrm{tr}(\omega_f(log\rho_f \otimes I)) - \mathrm{tr}(\omega_f(I \otimes \log\nu_i)) \\ &= \mathrm{tr}(\omega_i \log\omega_i) - \mathrm{tr}(\rho_f \log\rho_f) - \mathrm{tr}(\nu_f \log\nu_i) \end{aligned}$$

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$$\log\nu_i = -\beta H_{\mathcal{R}} - logZ$$

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$$\sigma = 0 \Longleftrightarrow \omega_f = \rho_f \otimes \nu_i \Longrightarrow \nu_f = \nu_i \Longrightarrow \Delta Q = 0 \Longrightarrow \Delta S = 0$$

$$\begin{aligned} 0 \leq \sigma = S(\omega_f|\rho_f \otimes \nu_i) &= \mathrm{tr}(\omega_f \log\omega_f) - \mathrm{tr}(\omega_f(log\rho_f \otimes I)) - \mathrm{tr}(\omega_f(I \otimes \log\nu_i)) \\ &= \mathrm{tr}(\omega_i \log\omega_i) - \mathrm{tr}(\rho_f \log\rho_f) - \mathrm{tr}(\nu_f \log\nu_i) \\ &= \mathrm{tr}(\rho_i \log\rho_i) + \mathrm{tr}(\nu_i \log\nu_i) - \mathrm{tr}(\rho_f \log\rho_f) - \mathrm{tr}(\nu_f \log\nu_i) \\ &= -\Delta S + \mathrm{tr}((\nu_i - \nu_f)\log\nu_i) \\ &= -\Delta S + \beta\Delta Q \end{aligned}$$

Landauer's Principle in statistical mechanics

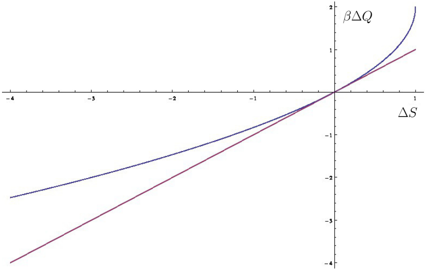
Remark 3. Landauer's bound is **not optimal** for finite dimensional reservoirs. Part of the analysis in [Reeb-Wolf '14] consists in refining it. A simple improvement, based on the well-known inequality

$$\|\omega - \nu\|_1^2 = \sup_{A \neq 0} \frac{|\mathrm{tr}\,((\omega - \nu)A)|^2}{\|A\|^2} \leq 2\,\mathrm{tr}\,(\omega(\log\omega - \log\nu))$$

is given by

$$\beta\Delta Q \geq \left(1 + \frac{1 - \sqrt{1 - \Delta S/S_0}}{1 + \sqrt{1 - \Delta S/S_0}}\right)\Delta S$$

where $S_0 = \beta^2\ell^2/8$ and $\ell = \mathrm{diam\,spec}(H_{\mathcal{R}})$



3. Algebraic framework – Abstract Landauer's Principle

[Reeb-Wolf '14] Conjecture: Landauer's Principle can probably be formulated within the general statistical mechanical framework of C^* and W^* dynamical systems and an equality version akin to (1) can possibly be proven.

Macroscopic reservoir should be idealized as infinitely extended

Familiar objects (Hamiltonians, density matrices,...) lose their meaning in the Thermodynamic Limit ...

...but other structures emerge (modular theory)

We shall work in the C^* setting, but the analysis extends to the W^* case

The algebraic framework I

C^*-dynamical system $(\mathcal{O}, \tau)$

- Unital C^*-algebra $\mathcal{O}$ (observables).
- Strongly continuous group $t \mapsto \tau^t = \mathrm{e}^{t\delta} \in \mathrm{Aut}(\mathcal{O})$ (Heisenberg dynamics).

State ω

- Positive linear functional $\omega : \mathcal{O} \to \mathbb{C}$ such that $\omega(\mathbb{1}) = 1$.
- Schrödinger evolution $\omega_t = \omega \circ \tau^t$.
- τ-invariant if $\omega_t = \omega$ for all t.

Thermal equilibrium state ω at inverse temperature $\beta = 1/T$

- (τ, β)-KMS state: $\omega(A\tau^{t+\mathrm{i}\beta}(B)) = \omega(\tau^t(B)A)$.
- τ-invariant.

Relative entropy of positive linear functionals

- Finite dimensional case: $S(\zeta_1|\zeta_2) = \mathrm{tr}(\zeta_1(\log \zeta_1 - \log \zeta_2))$.
- Extends to general C^*/W^* setting [Araki '75].
- $\zeta_1(\mathbb{1}) = \zeta_2(\mathbb{1}) \Rightarrow S(\zeta_1|\zeta_2) \in [0, +\infty]$, and $S(\zeta_1|\zeta_2) = 0$ iff $\zeta_1 = \zeta_2$.

Setup for Landauer's Principle

The system $\mathcal{S}$

- $\mathcal{O}_{\mathcal{S}} = \mathcal{B}(\mathcal{H}_{\mathcal{S}})$ finite dimensional C^*-algebra.
- Initial state $\rho_i(A) = \mathrm{tr}(\rho_i A)$.

The Thermal reservoir $\mathcal{R}$

- C^*-dynamical system $(\mathcal{O}_{\mathcal{R}}, \tau_{\mathcal{R}})$.
- $\tau_{\mathcal{R}}^t = \mathrm{e}^{t\delta_{\mathcal{R}}}$.
- Initial state is a $(\tau_{\mathcal{R}}, \beta)$-KMS state ν_i.
- Self-adjoint Liouvillean $L_{\mathcal{R}}$ implements $\tau_{\mathcal{R}}$ in the GNS representation of $\mathcal{O}_{\mathcal{R}}$ induced by ν_i.

Joint system $\mathcal{S} + \mathcal{R}$

- $\mathcal{O} = \mathcal{O}_{\mathcal{S}} \otimes \mathcal{O}_{\mathcal{R}}$.
- $\omega_i = \rho_i \otimes \nu_i$.
- Inner automorphism $\alpha_U(A) = U^* AU$, for some unitary $U \in \mathcal{O}$.
- State transformation $\omega_i \mapsto \omega_f = \omega_i \circ \alpha_U$.
- Reference "state" $\eta = \mathbb{1} \otimes \nu_i$.

Abstract form of Landauer's Principle

Set

$$\Delta S = S(\rho_i) - S(\rho_f), \qquad \Delta Q = -\mathrm{i}\omega_i(U^*\delta_{\mathcal{R}}(U))$$

Theorem 1

Assume that $U \in \mathrm{Dom}(\delta_{\mathcal{R}})$.

- $$\beta\Delta Q = \Delta S + \sigma, \qquad \sigma \geq 0$$
- If the point spectrum of the Liouvillean $L_{\mathcal{R}}$ is finite then $\sigma = 0$ iff $\Delta S = \beta\Delta Q = 0$. In this case $\nu_f = \nu_i$ and ρ_f is unitarily equivalent to ρ_i.

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Remark 1. Interpretation of ΔQ as dissipated heat requires more structure.

Remark 2. If $\mathcal{R}$ is confined then $\delta_{\mathcal{R}} = \mathrm{i}[H_{\mathcal{R}}, \cdot\,]$ and hence

$$\Delta Q = \omega_i(\alpha_U(H_{\mathcal{R}}) - H_{\mathcal{R}}) = \omega_f(H_{\mathcal{R}}) - \omega_i(H_{\mathcal{R}})$$

Moreover, the spectrum of $L_{\mathcal{R}}$ is finite $\Rightarrow$ Reeb-Wolf formulation of Landauer's Principle.

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Moreover, the spectrum of $L_{\mathcal{R}}$ is finite $\Rightarrow$ Reeb-Wolf formulation of Landauer's Principle.

Remark 3. If ν_i is ergodic, a natural assumption for a thermal reservoir, then 0 is the only eigenvalue of $L_{\mathcal{R}}$ and the second part of the theorem applies.

Remark 4. It is an interesting open problem to characterize all reservoirs for which this second part holds.

The algebraic framework II

GNS-Representation $(\mathcal{H}, \pi, \Omega)$

- $\mathcal{H}$ a Hilbert space.
- $\pi : \mathcal{O} \to \mathcal{B}(\mathcal{H})$ a $*$-morphism.
- $\Omega \in \mathcal{H}$ a vector such that $\pi(\mathcal{O})\Omega$ is dense in $\mathcal{H}$.
- $\eta(A) = (\Omega, \pi(A)\Omega)$.
- $\mathcal{N}$ set of η-normal states $A \mapsto \mathrm{tr}(\rho\pi(A))$ (ρ a density matrix on $\mathcal{H}$).
- $\pi \circ \tau_{\mathcal{R}}^t(A) = \mathrm{e}^{\mathrm{i}tL_{\mathcal{R}}}\pi(A)\mathrm{e}^{-\mathrm{i}tL_{\mathcal{R}}}$.

Ergodic/Mixing state

- ν_i is ergodic if, for all $\zeta \in \mathcal{N}$ and $A \in \mathcal{O}_{\mathcal{R}}$

$$\lim_{t\to\infty} \frac{1}{t} \int_0^t \zeta \circ \tau_{\mathcal{R}}^s(A)\, \mathrm{d}s = \nu_i(A)$$

- and mixing if

$$\lim_{t\to\infty} \zeta \circ \tau_{\mathcal{R}}^t(A) = \nu_i(A)$$

- Ergodicity/mixing can be characterized by spectral properties of the Liouvillean $L_{\mathcal{R}}$.

The entropy balance equation

Theorem (Perturbation of KMS structure) [Araki '73]

- If $K = K^* \in \mathcal{O}$, then $\delta_K = \delta_{\mathcal{R}} + \mathrm{i}[K, \cdot\,]$ generates a C^*-dynamical systems $(\mathcal{O}, \tau_K)$.
- There is a continuous map
$$\mathcal{O} \ni K = K^* \mapsto \omega_K$$
such that ω_K is the unique (τ_K, β)-KMS state in $\mathcal{N}$.
- For any positive linear functional ζ
$$S(\zeta|\omega_K) = S(\zeta|\eta) + \beta\zeta(K) + \log \|\mathrm{e}^{-\beta(L_{\mathcal{R}}+\pi(K))/2}\Omega\|^2$$

Combining Araki's theorem and Tomita-Takesaki's theory one can show

Theorem (Entropy balance) [Pusz-Woronowicz '78] [Ojima-Hasegawa-Ichiyanagi '88] [Jakšić-P '01]

If $U \in \mathrm{Dom}(\delta_{\mathcal{R}})$ and $\eta = I \otimes \nu_i$ (un-normalized $(\tau_{\mathcal{R}}, \beta)$-KMS) then
$$S(\omega \circ \alpha_U|\eta) = S(\omega|\eta) - \mathrm{i}\beta\omega(U^*\delta_{\mathcal{R}}(U))$$
for any state ω on $\mathcal{O}$ (both sides may be infinite).

The entropy balance equation

Proof of Theorem 1

Araki's perturbation theorem with $\tau = \tau_{\mathcal{R}}$, $\omega = \eta = I \otimes \nu_i$ and $K = -\beta^{-1} \log \rho$ yields

$$\omega_K = \rho \otimes \nu_i \Rightarrow S(\zeta|\rho \otimes \nu_i) = S(\zeta|\eta) - \zeta(\log \rho) \tag{2}$$

for any state ρ on $\mathcal{O}_{\mathcal{S}}$ and ζ on $\mathcal{O}$. In particular, with $\rho = \rho_i$ and $\zeta = \omega_i$ the LHS of (2) vanishes and

$$S(\omega_i|\eta) = \omega_i(\log \rho_i) = \mathrm{tr}(\rho_i \log \rho_i) = -S(\rho_i)$$

So we can write the entropy balance equation

$$S(\omega_f|\eta) - S(\omega_i|\eta) = -\mathrm{i}\beta\omega_i(U^*\delta_{\mathcal{R}}(U))$$

as

$$S(\rho_i) - S(\rho_f) + S(\omega_f|\eta) + S(\rho_f) = -\mathrm{i}\beta\omega_i(U^*\delta_{\mathcal{R}}(U))$$

which means

$$\Delta S + \sigma = \beta \Delta Q$$

whith the **entropy production** term

$$\sigma = S(\omega_f|\eta) + S(\rho_f)$$

Using again (2) with $\zeta = \omega_f$ and $\rho = \rho_f$ we finally get

$$\sigma = S(\omega_f|\rho_f \otimes \nu_i) \geq 0$$

with equality iff $\omega_f = \rho_f \otimes \nu_i$. The proof of the second part of the theorem relies on the spectral analysis of modular operators ($\Delta_{\omega_f|\omega_i} = \pi_{\omega_i}(U)\Delta_{\omega_i}\pi_{\omega_i}(U)^*$)

4. Tightness of Landauer's bound

- Can we identify ΔQ with dissipated heat ?
- Is Landauer's bound $\beta\Delta Q \geq \Delta S$ tight? i.e., how to achieve $\sigma = 0$ for a given state transition $\rho_i \to \rho_f$?
- What about non-faithful target states, e.g., $\rho_f = |\psi\rangle\langle\psi|$?

“Hamiltonian” dynamics

- Let $]0, t_f[\ni t \mapsto K(t) = K(t)^* \in \mathrm{Dom}(\delta_{\mathcal{R}})$ be a C^2 map with bounded first and second derivatives

τ_K^t is dynamics generated by $\delta_{\mathcal{R}} + \mathrm{i}[K(t), \cdot\,]$

- Interaction picture

$$\tau_K^t(A) = U_K(t)^* \tau_{\mathcal{R}}^t(A) U_K(t)$$

$U_K(t) \in \mathrm{Dom}(\delta_{\mathcal{R}})$ is the family of unitaries satisfying

$$\mathrm{i}\partial_t U_K(t) = \tau_{\mathcal{R}}^t(K(t)) U_K(t), \qquad U_K(0) = \mathbb{1}$$

- Since $\omega_i \circ \tau_K^t = \omega_i \circ \alpha_{\Gamma_K(t)}$ with $\Gamma_K(t) = \tau_{\mathcal{R}}^{-t}(U_K(t))$, Theorem 1 yields

$$\Delta S + \sigma = \beta \Delta Q$$

$$\Delta S = S(\rho_i) - S(\rho_{t_f}), \qquad \rho_{t_f} = \omega_i \circ \tau_K^{t_f}|_{\mathcal{O}_{\mathcal{S}}}$$

$$\Delta Q = -\mathrm{i}\omega_i(U_K(t_f)^* \delta_{\mathcal{R}}(U_K(t_f))), \qquad \sigma = S(\omega_i \circ \tau_K^{t_f} | \rho_{t_f} \otimes \nu_i)$$

- The energy balance is given by

$$\Delta Q + \left[\omega_i \circ \tau_K^t(K(t_f)) - \omega_i(K(0))\right] = \int_0^{t_f} \omega_i \circ \tau_K^t(\partial_s K(t))\, \mathrm{d}t$$

hence, if $K(0), K(t_f) \in \mathcal{O}_{\mathcal{S}}$ then ΔQ is the energy released in $\mathcal{R}$.

Adiabaticaly switched interactions

Set $t_f = 1$ and rescale $K_T(t) = K(t/T)$

$\tau^t_{K_T}$ dynamics generated by $\delta_{\mathcal{R}} + \mathrm{i}[K_T(t), \cdot\,]$ for $t \in [0, T]$

The previous analysis yields

$$\Delta S_T + \sigma_T = \beta \Delta Q_T$$

with

$$\Delta S_T = S(\rho_i) - S(\rho_T), \qquad \rho_T = \omega_i \circ \tau^T_{K_T}|_{\mathcal{O}_{\mathcal{S}}}$$

$$\Delta Q_T = -\mathrm{i}\omega_i(U_{K_T}(T)^* \delta_{\mathcal{R}}(U_{K_T}(T))), \qquad \sigma_T = S(\omega_i \circ \tau^T_{K_T}|\rho_T \otimes \nu_i)$$

In order to deal with the adiabatic limit $T \to \infty$ we shall make

Assumption P. ν_i is extremal $(\tau_{\mathcal{R}}, \beta)$-KMS state (pure phase)

Assumption A. For $\gamma \in]0, 1[$ the $(\tau_{K(\gamma)}, \beta)$-KMS state $\mu_{K(\gamma)}$ is ergodic for the dynamical system $(\mathcal{O}, \tau_{K(\gamma)})$

The adiabatic limit

Combining the gapless adiabatic theorem of [Avron-Elgart '99], [Teufel '01] and Araki's perturbation theory of KMS states leads to

Theorem 4

Suppose that Assumptions P and A hold. Then one has

$$\lim_{T\to\infty} \|\mu_{K(0)} \circ \tau_{K_T}^{\gamma T} - \mu_{K(\gamma)}\| = 0$$

for all $\gamma \in [0, 1]$.

A similar result was obtained and used by [Abou Salem-Fröhlich '05] to analyse quasi-static thermodynamic processes.

The adiabatic limit

According to the above adiabatic theorem, to implement a given state transition $\rho_i \to \rho_f$ in the limit $T \to \infty$ it suffices to supplement Assumption A with the boundary conditions

$$K_0 = -\beta^{-1} \log \rho_i, \qquad K_1 = -\beta^{-1} \log \rho_f$$

which imply $\mu_{K_0} = \rho_i \otimes \nu_i$ and $\mu_{K_1} = \rho_f \otimes \nu_i$ so that

$$\lim_{T\to\infty} \omega_i \circ \tau_{K_T}^T = \rho_f \otimes \nu_i$$

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It follows that

$$\Delta S = \lim_{T\to\infty} \Delta S_T = S(\rho_i) - S(\rho_f)$$

The energy balance equation, written as

$$\Delta Q_T = \int_0^1 \omega_i \circ \tau_{K_T}^{\gamma T}(\partial_\gamma K(\gamma))\,\mathrm{d}\gamma - \beta^{-1}\omega_i \circ \tau_{K_T}^{T}(\log \rho_f) + \beta^{-1}\omega_i(\log \rho_i)$$

further gives

$$\Delta Q = \lim_{T\to\infty} \Delta Q_T = \int_0^1 \mu_{K(\gamma)}(\partial_\gamma K(\gamma))\,\mathrm{d}\gamma + \beta^{-1}\Delta S$$

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which yields Landauer's Principle

$$\beta\Delta Q = \Delta S + \sigma, \qquad \sigma = \lim_{T\to\infty} \sigma_T = \beta \int_0^1 \mu_{K(\gamma)}(\partial_\gamma K(\gamma))\,\mathrm{d}\gamma \geq 0$$

The adiabatic limit

For this adiabatic process we expect saturation of the Landauer bound. Indeed,

Proposition 5

$$\sigma = 0$$

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Remark 1. The proof of the above proposition requires modular theory. It is a simple adaptation of the following elementary calculation which holds for finite reservoirs

$$\begin{aligned}\int_0^1 \mu_{K(\gamma)}(\partial_\gamma K(\gamma))\,\mathrm{d}\gamma &= \int_0^1 \frac{\mathrm{tr}\left(\mathrm{e}^{-\beta(H_{\mathcal{R}}+K(\gamma))}\partial_\gamma K(\gamma)\right)}{\mathrm{tr}\left(\mathrm{e}^{-\beta(H_{\mathcal{R}}+K(\gamma))}\right)}\,\mathrm{d}\gamma \\ &= -\frac{1}{\beta}\int_0^1 \partial_\gamma \log\mathrm{tr}\left(\mathrm{e}^{-\beta(H_{\mathcal{R}}+K(\gamma))}\right)\,\mathrm{d}\gamma \\ &= -\frac{1}{\beta}\left(\log\mathrm{tr}(\rho_f\otimes\nu_i) - \log\mathrm{tr}(\rho_i\otimes\nu_i)\right) = 0\end{aligned}$$

Note however that Theorem 4 and existence of $\lim_{T\to\infty}\sigma_T$ can not hold for finite reservoir.

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Remark 2. Non-faithful target states, e.g., $\rho_f = |\psi\rangle\langle\psi|$, are thermodynamically singular and cannot be reached by coupling $\mathcal{S}$ to a reservoir at non-zero temperature. Indeed, approximating ρ_f by faithful ρ one observes that $\sigma \to \infty$ as $\rho \to \rho_f$ for Hamiltonian dynamics and hence $\Delta Q \to \infty$. However, this instability does not occur in the adiabatic limit since $\sigma = 0$. Thus, adiabatic processes can reach a singular target state with arbitrary precision without producing entropy.

3. Summary

Summary

- The entropy balance relation is a model independent structural identity. It is tautological for confined systems

$$\begin{aligned} S(\omega \circ \alpha_U|\eta) - S(\omega|\eta) &= \mathrm{tr}\,(U\omega U^*(U \log \omega U^* - \log \eta) - \omega(\log \omega - \log \eta)) \\ &= \mathrm{tr}\,(\omega(U^* \log \eta U - \log \eta)) \\ &= -\beta \mathrm{tr}\,(\omega(U^* H_{\mathcal{R}} U - H_{\mathcal{R}})) \end{aligned}$$

  It follows from Araki's perturbation theory of KMS structure for extended systems. It plays a central role in the analysis of the second law in open quantum systems. It provides a natural approach to LP in quantum statistical mechanics with precise hypotheses that also set limits to its validity ([Allahverdian-Nieuwenhuizen '01], [Alicki '14]).

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- These ergodic properties have been established for various models [Botvich-Malyshev '83, Aizenstadt-Malyshev '87, Jakšić-P '96, Bach-Fröhlich-Sigal '00, Jakšić-P '02, Dereziński-Jakšić '03, Fröhlich-Merkli-Ueltschi '03, Aschbacher-Jakšić-Pautrat-P '06, Jakšić-Ogata-P '06, Merkli-Mück-Sigal '07, de Roeck-Kupianien '11]. Further progress in this direction requires novel ideas and techniques in the study of the Hamiltonian dynamics of extended systems.

Thank you !